# The Separate Path and the Well Traveled Road

Barry Garelick*

Much has been written about the debate on how best to teach math to students in the K12 grades—a debate often referred to as the "math wars". I have written much about it myself, and since the debate shows no signs of easing, I continue to have reasons to keep writing about it. While the debate is complex, the following two math problems provide a glimpse of two opposing sides:

Problem 1: How many boxes would be needed to pack and ship one million books collected in a school-based book drive? In this problem the size of the books is unknown and varied, and the size of the boxes is not stated.

Problem 2: Two boys canoeing on a lake hit a rock where the lake joins a river. One boy is injured and it is critical to get a doctor to him as quickly as possible. Two doctors live nearby: one up-river and the other across the lake, both equidistant from the boys. The unhurt boy has to fetch a doctor and return to the spot. Is it quicker for him to row up the river and back, or go across the lake and back, assuming he rows at the same constant rate of speed in both cases?

The first problem is representative of a thought-world inhabited by education schools and much of the education establishment. The second problem is held in disdain by the same, but favored by a group of educators and math oriented people who for lack of a better term are called "traditionalists".

In the spirit of full disclosure, I fit in with the latter group. I also plan to teach mathematics when I retire soon. I have a more than brushing acquaintance with the thinking of the education establishment side of the war from classes I have taken over the last few years at education school

I have heard various education school professors distinguish between "exercises" and "solving problems". Textbook problems are thought of as "exercises" rather than "problems" because they are not real world and therefore, in their view, not relevant to most students. Opponents of such problems contend that they contain the quantities students need to solve the problems and therefore do not require students to make value judgments. Such criticisms of traditional approaches in mathematics have led to an approach that I call "The Separate Path" because it generally takes students on a path they have never seen before.

The first problem presented above is an example of a "separate path" type problem which appeared in a paper called Teacher as designer: A framework for teacher analysis of mathematical model-eliciting activities, by Hjalmarson and Dufies Dux. The problem is called "The Million Book Challenge". While it may be engaging, students will generally lack the skills required to solve such a problem, such as knowledge of proper experimental approaches, systematic and random errors, organizational skills, and validation and verification.

Problems such as the million book challenge are predicated on the idea that by repeatedly confronting students with new situations as well as problems with which they have little to no experience, they will develop the "habits of mind" that lead to mathematical reasoning. The open-endedness of the problem is seen as a means to engage students in the "process" of critical thinking. In my opinion, however, such approach is like learning German by practicing particular sentence constructions in English (e.g., "I know that he the book read has") in the hopes of building up a structure that then only needs vocabulary to complete the learning process.

Let’s turn now to the second problem—a problem appropriate for an honors algebra 1, or regular algebra 2 class. Unlike the million book challenge, it allows students to rely on prior knowledge, it is well-defined, and has specific mathematical goals. I think of problems such as the canoe problem as going down a well traveled road—you know where it is taking you even though it may have a few twists and turns and detours in unfamiliar territory.

While problems such as the million book challenge are touted as showing how the real world describes what’s going on more than mathematics alone can do, the canoe problem challenges that view. It demonstrates that mathematics is needed to describe what is going on—and contradicts what one intuitively believes.

Many students will assume that either route will take the same amount of time. They reason that if the boy travels upstream and downstream for the same distance at the same rate of speed, the amount the canoe is slowed by the current when travelling upstream is cancelled by the additional speed the current imparts when travelling downstream. But the mathematics shows otherwise.

Letting D be the distance to each doctor, R the rate of speed of the canoe and C the speed of the river current, we know that since distance = rate x time, the time to go across the lake and back is $2D/R$. The time to go up the river and back is $D/(R-C) + D/(R+C)$. If the time to cross the lake is equal to the time to go up river and back, then $2D/R$ would equal $2DR/(R2 - C2)$ .

Algebraic exploration shows, however, that $2D/R$ is less than $2DR/(R2 - C2)$; thus it takes less time to go across the lake and back than to go up the river and back. Note that this problem does not provide any values for distance, speed of boat or speed of current. Students must model the situation using symbols, and apply their knowledge of expressing time as a function of rate and speed. It is also interesting that this problem does not lend itself to a “plug-in” solution.

Also, the proof that $2D/R$ is less than $2DR/(R2 - C2)$ is not easy and it may well be that students will have some difficulty with this part of the problem. Proofs of inequalities do not easily lend themselves to algorithmic solutions--in fact, they require the critical thinking and analytical skills that many believe problems like the Million Book Challenge develop. To do this problem in the later part of an Algebra 1 course or in Algebra 2 would not introduce anything students haven’t learned. The student has the tools with which to do it, though this doesn’t mean that all students will be able to solve it. Some students may find such problems very difficult. But according to Vern Williams, a middle school math teacher, and who served as a member of the President's National Mathematics Advisory Panel, “By taking math that has been taught to them and attempting to solve difficult problems, they will discover relationships between content and methods that they already have in their arsenal even if they don't solve the problem or arrive at the correct answer.”

The issue of separate path type problems versus the well traveled path has particular significance in light of the recent interest in developing national assessments for math. Critics of the well-traveled road approach to math tend to believe that assessments should evaluate the "critical thinking" skills of students rather than having students solve "exercises" that lend themselves to applications of previously learned procedures. Many such critics also believe that students in other countries that surpass the US in math on international tests are being taught only how to take tests. Ironically, the problem with a test that emphasizes the separate path type of problems is that it accommodates students' learning how to answer open-ended questions. The U.S. may then achieve high test scores, but when process trumps content, what mathematics are students really learning? In the end the problems which students in Singapore, Hong Kong and Japan excel at solving are still likely to be off the script for many US students.

In a world in which problems that have a unique answer obtained through systematic application of mathematical skills and principles are deemed "mere exercises", students are heading down a separate path approach to learning that leads at best to math appreciation, and at worst, a turn-off to the subject. If, however, students are taught the skills and concepts necessary to solve well-defined and challenging problems, they will learn to surmount what a disheartening number of U.S. students now consider to be insurmountable.

Citation: Garelick, B. (2010). The Separate Path and the Well Traveled Road? *Nonpartisan Education Review / Essays, 6*(2). Retrieved [date] from http://www.npe.ednews.org/Review/Essays/v6n2.pdf

---

*. Barry Garelick is an analyst for the Environmental Protection Agency in Washington DC. He majored in mathematics at the University of Michigan, and is a co-founder of the US Coalition for World Class Math, a group of mathematically literate parents and educators who want excellence in mathematics education in the United States.