

Section C

The Inconvenient Truth:

**Reform Math is Firmly Rooted in Profound Anti-intellectualism,
Racism, Sexism, and Elitism**

**Reform Math is Abysmally Wrong and Unjust,
Disproportionally Victimizing Disadvantaged Students**

CMF 2021:

In California in the years 2004–2014, 32 percent of Asian American students were in gifted programs compared with 8 percent of White students, 4 percent of Black students, and 3 percent of Latinx students.

It is common for people to claim that avoiding aspects of race, culture, gender, or other characteristics as they teach mathematics, means they are being equitable; but the evolution of mathematics in educational settings has resulted in **dramatic inequities for students of color, girls, and students from low income homes.**

These inequalities include not only **access** to high-quality curriculum and resources, but also to instruction that appropriately leverages students' diverse knowledge bases, identities, and experiences for both learning and developing a sense of belonging to mathematics.

Unlike their peers, vast Black, Latinx, and Native American students who rely exclusively on school education have no access to supplemental resources to make up for their severe shortfalls in real math knowledge and skills caused by reform math.

They have full academic potentials. But reform math shortchanges them.

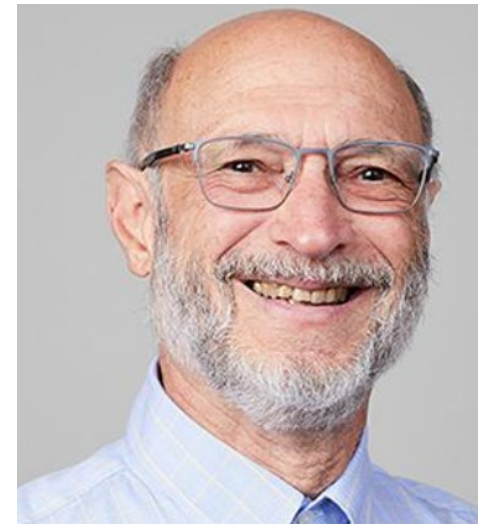
Denying disadvantaged kids the chance of pursuing a better life through good education, reform math is the worst social injustice and a most evil crime on earth!

- “If all students do not have an opportunity to learn this mathematics, **we face the danger of creating an intellectual elite and a polarized society**. The image of a society in which a few have the mathematical knowledge needed for the control of economic and scientific developments is not consistent either with the values of a just democratic system or with its economic needs.”

-- The 1989 NTCM Standards

- Alan Schoenfeld describes the traditional curriculum as **elitist** and portrays the math wars as a battle between equality and elitism: **"the traditional curriculum bore the recognizable traces of its elitist ancestry: the traditional curriculum was a vehicle for . . . the perpetuation of privilege. . .** Thus the Standards could be seen as a threat to the current social order. . the traditional curriculum, with its filtering mechanisms and high dropout and failure rates (**especially for certain minority groups**) has had the effect of putting and keeping certain groups 'in their place'." "lack of access to mathematics is a barrier – a barrier that leaves people socially and economically disfranchised."

Traditional Math vs.
Reform Math: Which
one is truly rooted in
racism/elitism/sexism?
Which one is
perpetuating the
academic achievement
gap?



CMF 2021: Mathematics education in the United States was initially structured for a narrow purpose: to prepare privileged, young, white men for entrance into elite colleges. While instruction has shifted toward learning with understanding, and the field increasingly attends to issues of equity and access.

Response: Do you agree that traditional algebra, geometry, and calculus are pivotal to the development of science, technology, and economy? If traditional math is a privilege, why not strive to help all children achieve this privilege? Why instead water down math education by redefining content, methodology, and assessment of K-12 math, which stands to shortchange all students and victimize especially the disadvantaged kids?

CMF 2021: Mathematics pathways must open mathematics to all students, eliminating option-limiting tracking. An important goal of this framework is to replace ideas of innate mathematics “talent” and “giftedness” with the recognition that every student is on a growth pathway.

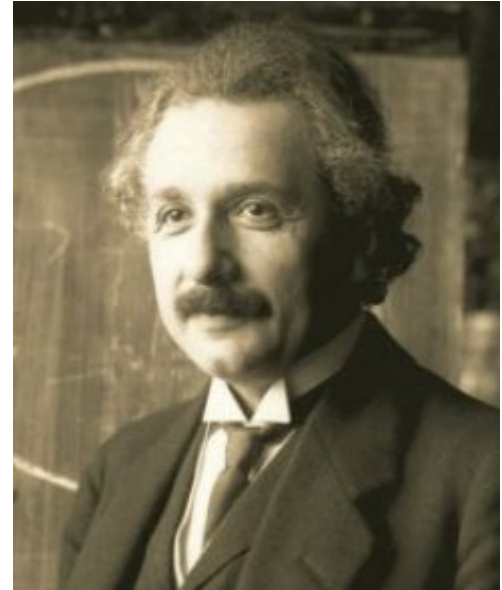
Response: If schools teach traditional math from first grade on, keeping up curricular rigor, focus, and coherence, giving corrective feedback to help students learn from their mistakes, and abandoning the spiraling content arrangement, then tracking may be unnecessary in elementary and middle schools. This is the common practice in the high-performing regions and countries.

China's Math Education Achievement Disproves Racism Based Reform Math

In the early 20th century, racial and gender prejudices advanced by the pioneer progressive educators were common in America and abroad. In the diaries about his visit to China in 1922, Einstein jotted down such laments: **“The Chinese are incapable of being trained to think logically and that they specifically have no talent for mathematics;” “Even the children are spiritless and look obtuse.”**

When Einstein's diaries were disclosed in 2018, the voices over the Chinese media overwhelmingly defended him: Einstein simply depicted the true state of the Chinese masses at that time.

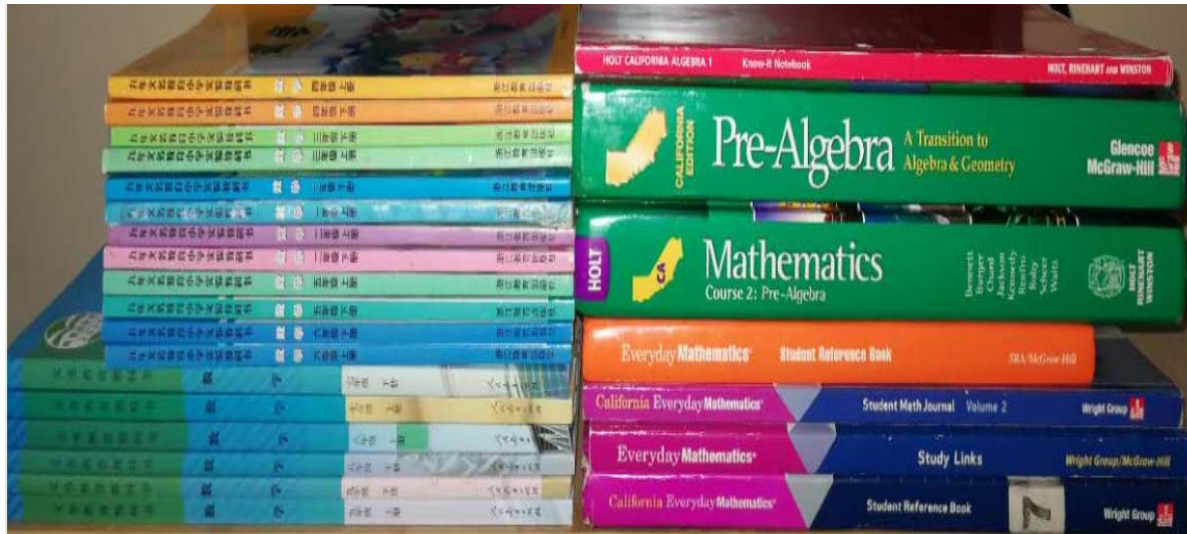
Shortly after World War II, the Chinese people established an Orwellian collectivism system and ended up with equality in abject poverty. A silver lining of this era was China's adoption of the Russian K-12 math programs, developed under the supervision of pre-eminent mathematician Andrey Kolmogorov, in the mid-1950s. Since China opened up in the late 1970s, its solid K-12 education has produced millions of scientists, engineers, and technicians, the brainpower essential to China's rapid industrialization and the **greatest poverty-reduction achievement in human history.**



China's Math Education Achievement Disproves Racism Based Reform Math

To the left of the attached image is a complete set of Chinese 1st-9th grade math textbooks--thin and small (\$1-2 each), but rigorous, coherent, and focused; they turned a once hopelessly uneducated people into a top performer in international math assessments.

I hope this anecdote conveys two messages to you: First, racism and sexism based reform math is profoundly wrong and unjust; Second, rigorous programs using clear and coherent textbooks from early grades on are crucial for successful math education.



All humans, regardless of race, gender, or location, are capable of learning traditional math. History has proved racism and sexism based reform math is profoundly wrong and unjust.

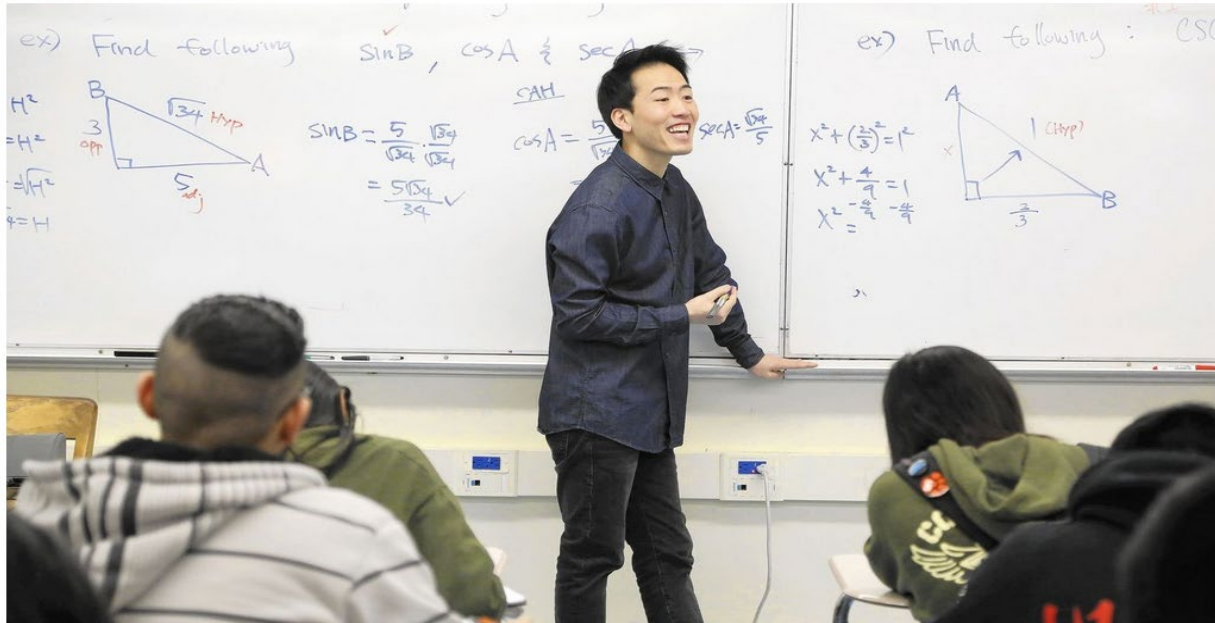
Why have America's disadvantaged kids persistently underachieved in math? Because they have been taught Fuzzy Math since first grade and, unlike their peers, they don't have resources to remedy their shortfalls in basic knowledge and skills!!

Disadvantaged Minorities Can Achieve in Real Math

How a Lincoln High teacher gets all his students to pass the AP Calculus exam



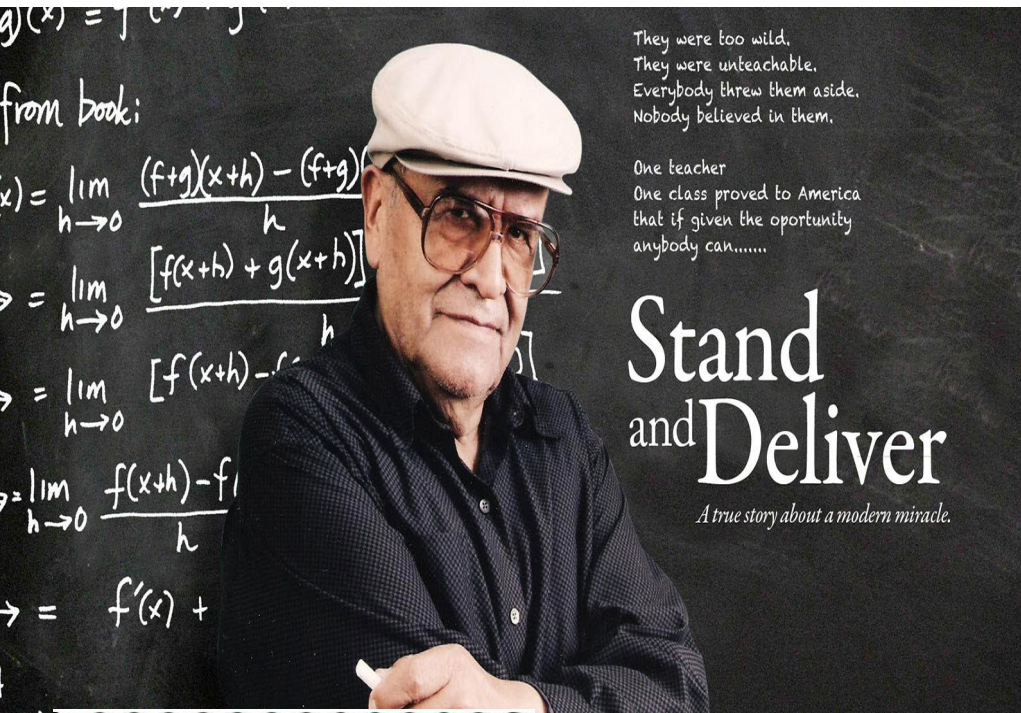
By STEVE LOPEZ FEB 03, 2016 | 1:30 AM



Abraham Lincoln High School senior Cedrick Argueta received a perfect score on the AP Calculus exam.

In 2016, at Lincoln High, Los Angeles, a school with 80 percent of Latino students, math teacher Anthony Yom got his whole class to pass the AP Calculus exam, and a student named Cedrick Argueta was one of the twelve perfect-score winners in the world.

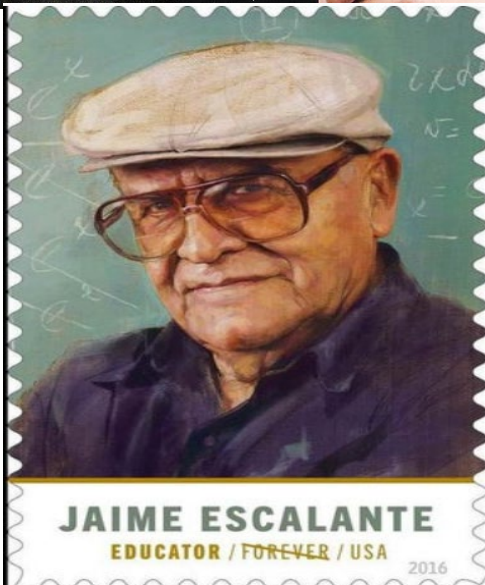
Disadvantaged Minorities Can Achieve in Real Math



In the 1980s, Jaime Escalante, a math teacher at an East Los Angeles high school in an impoverished neighborhood, got hundreds of his students—sons and daughters of day laborers, seamstresses, house cleaners—to pass the AP Calculus exam, and many of them have accomplished remarkably well in college and in their later careers. [Sergio Valdez is one of them.](#)



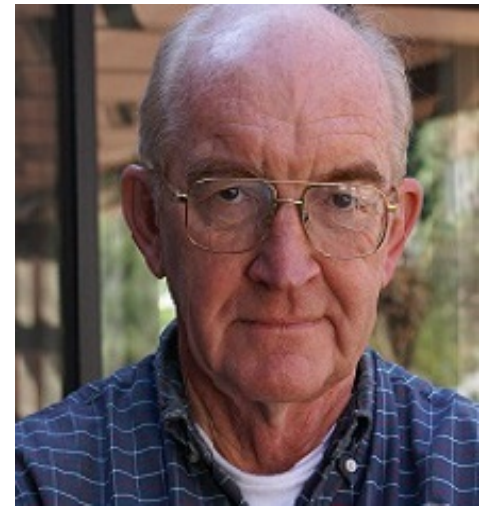
EAST LA NATIVE, JAIME ESCALANTE'S STUDENT LEADS TEAM IN NASA'S SUCCESSFUL LANDING OF PERSEVERANCE ROVER ON MARS



Stand and Deliver Revisited, by Jerry Jesness

- Conventional pedagogical wisdom holds that the poor, the disadvantaged, and the **"culturally different"** are a fragile lot, and that the academic rigor usually found only in elite suburban or private schools would **frustrate them, crushing their self-esteem**. The teachers and administrators that I interviewed did not find this to be true of Garfield students.
- Wayne Bishop, a professor of mathematics and computer science at California State University at Los Angeles, notes that Escalante's top students generally did not attend Cal State. Those who scored fours and fives on the A.P. calculus tests were at schools like MIT, Harvard, Yale, Berkeley, USC, and UCLA. For the most part, Escalante grads who went to Cal State-L.A. were those who scored ones and twos, with an occasional three, or those who worked hard in algebra and geometry in the hope of getting into calculus class but fell short.
- Bishop observes that these students usually required no remedial math, and that many of them became top students at the college. **The moral is that it is better to lose in the Olympics than to win in Little League, even for those whose parents make less than \$20,000 per year.**
- To Ms Colaizzi and the Pittsburgh Public Schools BoE, by Wayne Bishop

Although such absolutes are hard to substantiate objectively, it is possible that this is the worst high school mathematics curriculum that has ever been written...**districtwide approval of IMP (Interactive Math) would be nothing short of immoral. The opportunity of upward mobility through education lost for thousands of children.**



Wayne Bishop, former Chair of department of math/CS at CSU Los Angeles, was mentor to Jaime Escalante and publicly honored by him in the premiere showing of *Stand and Deliver*.

Disadvantaged Minorities Can Achieve in Real Math

2019 NEW YORK STATE EXAMS

Scaling Excellence

Once again, Success Academy scholars topped the charts of New York State schools: Number 1 in the state! As the state's seventh-largest school district,* Success enrolls 17,000 students across 45 schools, educating about one in 60 public school students in New York City. On this year's state exams, our test-takers surpassed their more affluent peers across the state, with 90% passing English Language Arts and 99% passing math.

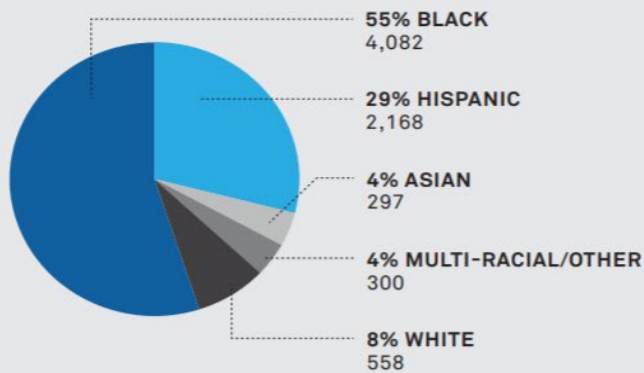
DISTRICT	OVERALL MATH PASS RATE	OVERALL ELA PASS RATE	DEMOGRAPHICS
#1 Success Academy	99%	90%	Enrollment: 17,000 Household Income: \$49,800 94% Black and Hispanic Students



GPA: 3.90

College Bound:
Rochester Institute

This Year's Test-Takers



Test-takers included:

19% students with IEPs
3% English language learners
6% homeless students



How Not to Teach Math, by Matthew Clavel

- The repudiation of skills in Fuzzy Math also encourages **a detrimental overreliance on calculators**. The use of these gadgets to replace mental computation raises concerns about learning skills for all school children. According to a 2000 Brookings Institute study, fourth graders who used calculators every day were likely to do worse in math than other students. **But it's minority kids like those in my class who are turning to calculators the most.** The Brookings study reports that **half of all black school children used calculators every day**, compared with 27 percent of white school kids.
- Regrettably, in the heavily bureaucratized public schools, bad results do not necessarily lead to re-evaluation. **Fuzzy Math, cooperative learning, and myriad other educational fads** are the pet projects of **very influential, tenured university "experts," who fiercely protect their theoretical turf, in teachers colleges and among school administrators**. If test scores seem to rise thanks to Fuzzy Math, great: campus enthusiasts will tout the results. If they stagnate or fall, the theoreticians will find ways to poke holes in any critical study that blames the theory.
- Nor will school bureaucrats usually be quick to get rid of a deeply flawed curriculum. After all, if the "experts" say Fuzzy Math is the way to go—and school administrators are loath to challenge the experts—then the problem must be in how teachers are implementing the theory, not in the theory itself.
- **The inner-city students subjected to this curriculum will be the real losers.** What will happen to kids who never adequately learned basic operations like long division—or even their times tables? How will they succeed in the knowledge-based twenty-first century economy? **Most of them won't have parents who can afford math tutors to help them catch up. My guess is that most of these kids will never get the remedial education they need**, and that we'll just brush another catastrophe under the rug.

Math Wars, Wall Street Journal Editorial, 2000

- **Reinventing math is an old tradition in this country.** It has been around at least since the 1960s, when the inimitable Tom Lehrer mocked the New Math in Berkeley cafes. Today the original New Math is old hat, but many folks in the education world are hawking yet another reform. It is known by names like "Connected Math," or "Everyday Math."
- Next comes Connected Math, another panel favorite. **It too skips or glosses over crucial skills.** Example: The division of fractions, an immutable prerequisite for algebra, is absent from its middle-school curriculum. In shutting the door to algebra, David Klein of Cal State Northridge points out ...
- Everyday Math **ensures juvenile dependency to calculators** by endorsing their use from kindergarten. Rather than teach long division, the program devotes substantial time to that important area of math study, self-esteem.
- And then move on to the main question: Why? The reason for the New New Math, as for many other curriculum reforms, is that teachers, school administrators and their **unions are tired of being blamed for statistical declines and poor student performances.**
- New Mathie and federal panel member Steven Leinwand explains: "It's time to recognize that, for many students, real mathematical power, on the one hand, and facility with multidigit, pencil-and-paper computational algorithms, on the other, are mutually exclusive." Or, **as Professor Klein translates: "Underlying their programs is an assumption that minorities and women are too dumb to learn real mathematics."**
- **New Math will take its casualties, especially among the poor**, adding to the already mounting costs of the decline in national educational standards.

- William C. Bagley and Antonio Gramsci who explained in the 1930's **why the new educational ideas would lead to greater social injustice.**

In 1932, when in prison in Italy, the Communist intellectual Antonio Gramsci wrote the following:

*The new concept of schools is in its **romantic phase**, in which **the replacement of 'mechanical' by 'natural' methods has become unhealthily exaggerated** ... Previously pupils at least acquired a certain baggage of concrete facts. Now there will no longer be any baggage to put in order... The most paradoxical aspect of it all is that this new type of school is advocated as being democratic, while in fact **it is destined not merely to perpetuate social differences** but crystallize them in Chinese complexities.*

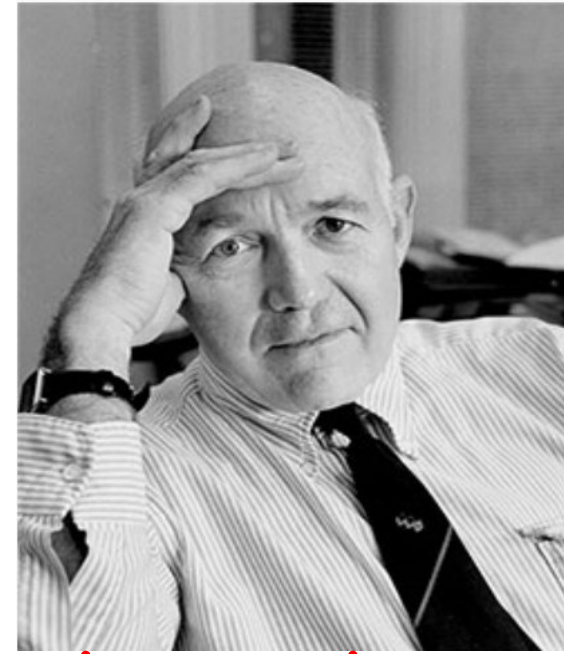
- "...I think we are sometimes guilty of not teaching to the rigor of those courses. ... **We sometimes lower the bar because we want to make sure everyone gets over it.**"
-- Donald Pittman, Chicago Public Schools chief officer for high schools

- [**The Northridge Chronicles: A Virtual Play**](#)

They seek to deny knowledge and **their solution is to "dumb down."** One of our colleagues stated, while discussing the Harvard text in the mailroom:

We have inferior students and we must teach them in an inferior manner.

James Samuel Coleman



- Yet the phrase "*Developmentally Appropriate Practice*," has been very effective politically. It has played on our love and solicitude for young children. It is used as a kind of conversation stopper. If one is told that an educational recommendation is "*developmentally inappropriate*," one is supposed to retreat and remove the offending item from the early curriculum. But this retreat has to stop. We must stand up to unsupported rhetorical bullying, and rely on the people who know the research. To cave in to intimidating rhetoric is to harm our children, not help them. The romantic doctrine of NAEYC is **wasting minds and perpetuating social inequities**.

A rich and coherent early curriculum improves performance for all, but improves it most for disadvantaged students, thus narrowing the equity gap.

As the late, great James Coleman showed, it is ineffective early schooling coupled with economic class, not with race or ethnicity, that causes the academic achievement-gap. The intention is to insure caring treatment for young children, yet the ultimate effect of the doctrine is to cause social harm. To withhold demanding content **from young children between preschool and third grade** has an effect which is quite different from the one intended. It leaves advantaged children (who get knowledge at home) with boring pablum, and it **condemns disadvantaged children to a permanent educational handicap that grows worse over time**.

If anyone is racist or sexist, it is those who claim that women and minorities are unable to deal with traditional mathematics.

-- Barry Simon

Former mathematics chair and IBM Professor of Mathematics and Theoretical Physics at Caltech



Finding the Answers In Drills and Rigor, by E. D. Hirsch, Jr.

The main cause of inequality in American schools ... has been the dominance of the progressive-education tradition, which has seriously misconceived itself as the guardian of social progress and democratic ideals. ... If we are lucky, the end of the 1990s will mark the end of spurious connections between educational ideas and political affiliations. ... Teaching a curriculum that produces high literacy for all is a potent way of fostering the egalitarian goal of democratic education. But before we can advance toward that goal on a broad front, many progressivist ideas will have to be discarded.

I think you would get consensus from mainstream science on the following prediction: **that if we bring all children to readiness in the early grades, then the achievement of excellence and equity in later grades will begin to be possible.** If I were a member of your Board, I would to begin to shift rather large resources but only into academically effective, really effective very-early education. In due course, such a policy would loosen up a lot of remedial money that could be spent on improving very early education still more. **Overcoming the inadequacies of early education is the most effective way of preventing the inadequacies that exist at 12th grade.**

CMF 2021:

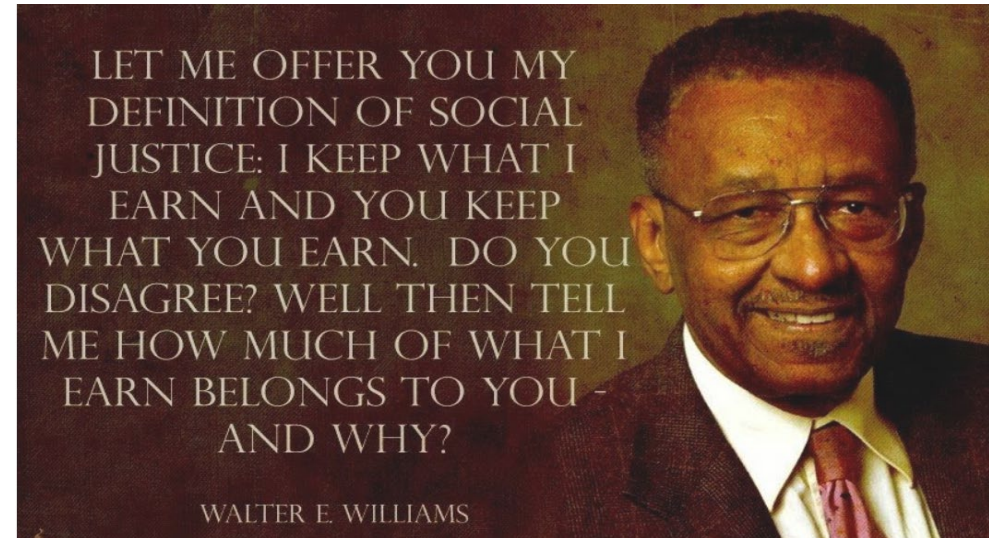
Culturally Responsive Mathematics Education, for example, emphasizes active, collaborative communities of learners engaged in mathematical explorations through meaningful and personally-relevant social contexts.

The belief that “I treat everyone the same” is insufficient: Active efforts in mathematics teaching are required in order to counter the cultural forces that have led to and continue to **perpetuate current inequities.**

A “color-blind” approach allows such systemic inequities to continue.

Walter Williams, "Suffer No Fools":

“I often tell people that, I am very very happy that I got virtually all of my education **before it became fashionable for white people to like black people.** So what that meant is when I got a C, it was an honest-to-God C; when I got an A, it was an honest-to-God A. They weren't practicing affirmative action, and they didn't give a damn about my self-esteem.”



“The author of 13 books, dozens of academic papers and countless popular essays, Walter was a scholar’s scholar. ” -- Donald J. Boudreaux

Section D

The Incalculable Cost of Calculators

One of the most deleterious messages from math reformers is that computational fluency is an obsolete skill, and students should focus on thinking and understanding instead.

The truth is fluency in arithmetic facts and skills is critically important in working on all levels of math. If there could be only one suggestion on improving the math performance of American students, it is to restrict the usage of calculators and other calculating facilities to a minimum during K-10 grades or even K-12 grades.

- [Kick Calculators out of Class](#) by David Gelernter

They should be banned from American elementary schools. ... The calculator subtly undermines the whole math curriculum.

The basics are not simply computation, although that is how detractors have tried to confine the debate, seeking to portray critics of the mathematics education establishment as advocates of repetitive and mindless computational exercises.



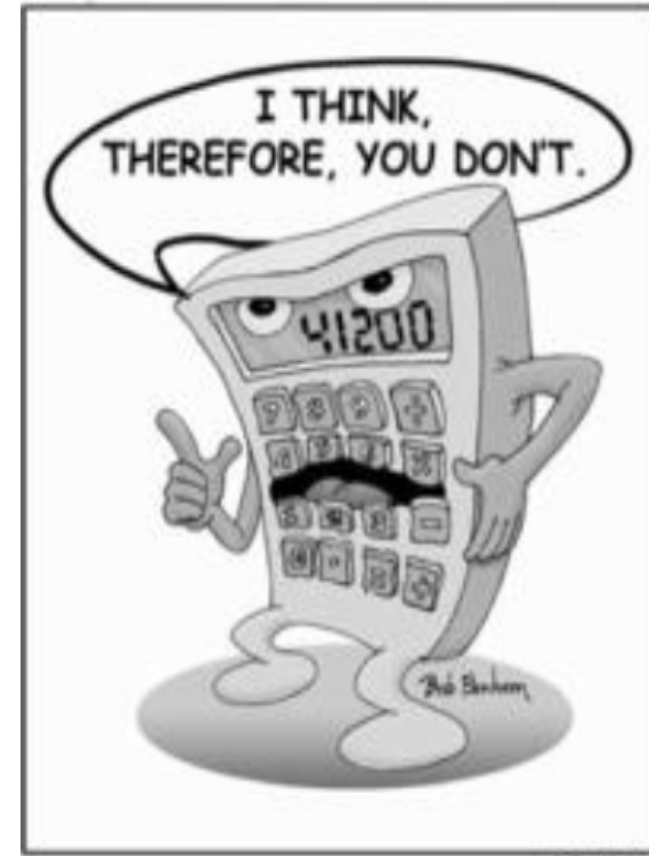
- [What to do about Canada's declining math scores](#), by Anna Stokke

Early achievement in mathematics is a strong predictor – even more so than reading skills – of later academic achievement, financial success, and future career options (Charette and Meng 1998; Duncan et al. 2007; Duncan 2011; Romano et al. 2010). **Students who struggle early in math struggle later on**: sixth graders who fail math have less than a one-in-five chance of starting twelfth grade on time, and only 19 percent graduate on time or within a year (Balfanz 2007)

There is plenty of evidence that **automatic recall of number facts is extremely important** for success in math (Price et al. 2013; Qin et al. 2014). Studies confirm – after controlling for various factors such as general intellectual ability, working memory, family income and education – that **fluency with fraction arithmetic, without the use of calculators or technology, in early and middle-years** students predicts student knowledge of algebra and later high-school math achievement (Bailey et al. 2012; Siegler et al. 2012)

Calculators

- [Why Johnny Can't Add Without a Calculator](#) by Konstantin Kakaes, *Slate*, June 25, 2012. "Technology is doing to math education what industrial agriculture did to food: making it efficient, monotonous, and low-quality."
- [Calculating the Cost of Calculators](#) by Lance T. Izumi, Capital Ideas, December 21, 2000. "A September 2000 Brookings Institution study found that calculator use decreases student math achievement. Analyzing national test data, Brookings concluded that **students who used calculators every day scored lower than students who used the devices less frequently**. Given this finding, it is disturbing that Brookings also found that while only 27 percent of white students used calculators daily, half of African-American students made daily use of calculators. Yet, despite such evidence, university schools of education, which place so much emphasis on the learning 'process,' actively promote the use of



Used with permission

• Math professors 93-to-0 in supporting basic math

Stephen Wilson, professor of mathematics at Johns Hopkins University, asked a number of mathematicians their thoughts on the following statement:

"In order to succeed at freshmen mathematics at my college/university, it is important to have knowledge of and facility with basic arithmetic algorithms, e.g. multiplication, division, fractions, decimals, and algebra, (without having to rely on a calculator)."

The response was overwhelming and unanimous:

93 mathematicians agreed with the statement, and NONE disagreed!

• K-12 Calculator Usage and College Grades

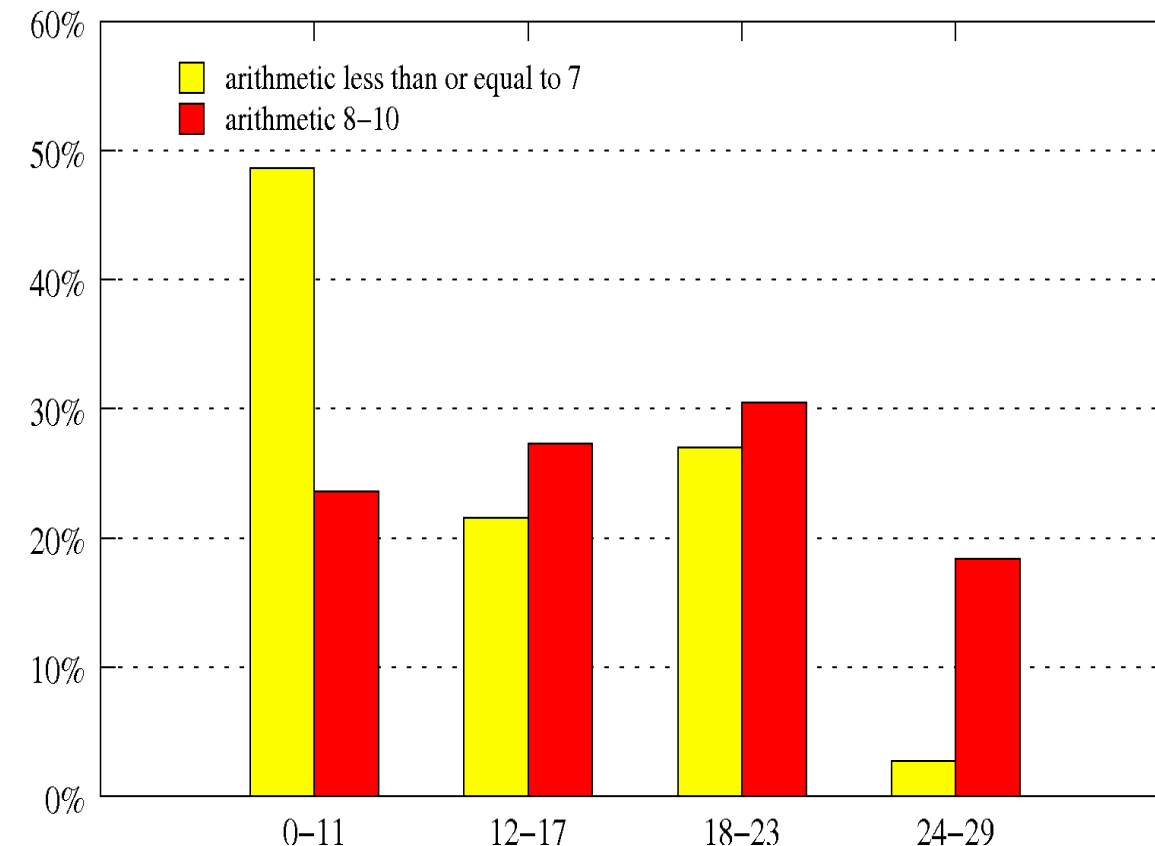
by W. Stephen Wilson and Daniel Q. Naiman

This study concludes that students in the math courses at Johns Hopkins University who had been encouraged to use calculators in K-12 had lower grades than those who weren't.



W. Steve Wilson gave the 239 students in his Calculus III class a 10-point arithmetic test in the first lesson and a 30-point calculus test in the end. Test scores show students with poorer arithmetic skills scored lower in calculus.

Figure 1: Arithmetic test scores and final exam scores



- [Just Say No To Calculators](#) by David Klein

One of the most debilitating fads to sweep American public schools in the last decade has been the heavy use of calculators, especially in elementary schools. According to the Third International Mathematics and Science Study, or TIMSS, use of calculators in U.S. fourth-grade mathematics classes is about twice the international average. In six of the seven top-scoring nations in the study, teachers of 85 percent or more of the students report that students never use calculators in class. ... Even universities are forced to run remedial math classes at unprecedented levels, including classes in arithmetic for entering freshmen. This intolerable state of affairs can be laid at the doorstep of the uncritical use of calculators in elementary schools.

- [Computation Skills, Calculators, and Achievement Gaps: An Analysis of NAEP Items](#) by Tom Loveless

This valuable paper examines national trends in computation skills, investigates whether allowing calculators on NAEP items produces significantly different results compared to not allowing calculators, and analyzes the impact of allowing calculators on the performance gaps among black, white, and Hispanic students. It concludes: “If students are only able to compute accurately with calculators -- or if their computational skills are so weak that only the simplest of calculations can be made -- **then students are doomed to solving only trivial mathematical problems.**”

- [Calculators in High School Linear Algebra](#), by Ralph Raimi

The author describes this article as telling "how recent NCTM-approved advances in mechanized pedagogy can speed up the process of not learning anything.“

- [Math Teachers Debate The Use Of High-Powered Calculators](#) by Lisa Black

A hand-held calculator that can solve brain-numbing algebra equations within seconds has high school math teachers divided over whether it will make algebra more accessible or rob students of basic skills.

- [Paper-and-Pencil Math](#) by Richard H. Escobales Jr.

Mastery of addition and the other algorithms of basic arithmetic act as a flashlight, allowing the young student to move freely about in the world of numbers and basic numeric operations. Without such mastery a young student is condemned to move about blindly in this intriguing unknown world of numbers.

- [Of Culture, Calculators, Math Anxiety And How We Learn](#) UniSci News

Better performance on the complex arithmetic problems in a study was associated with lower reported calculator use in elementary and secondary school. Complex arithmetic places special demands on short-term memory skills that simple arithmetic usually does not, because complex arithmetic involves operations such as carrying, borrowing and place-keeping. This is demanding mental juggling for most people's short-term working memory processes. Using a calculator might restrict the level of expertise achieved with respect to short-term memory skills for complex arithmetic.

- [Ditch the Calculators: Letting Children Punch Numbers Into a Machine Does Not Add Up To Learning Math](#)

by Diane Hunsaker

Some teachers argue that calculators let students concentrate on how to solve problems instead of getting bogged down with tedious computations. ... Some of my elementary-school children look at a word problem and instantly guess that adding is the correct approach. When I suggest that they solve the problem this way without a calculator, they usually pause and think before continuing. **A student is much more likely to minimize his work by reflecting on the problem first if he doesn't have a calculator in his hand. ... A student who learns to manipulate numbers mentally can focus on how to attack a problem and then complete the actual computations easily.** He will also have a much better idea of what the answer should be, since experience has taught him 'number sense,' or the relationship between numbers. **A student who has grown up with a calculator will struggle with both strategies and computations.**

Address to California State Board of Education, by E.D. Hirsch, Jr.

- **The NCTM group stresses conceptual understanding over mindless drill and practice, while the dissident group stresses the need for drill and practice leading to mastery.** To resolve the issue, which researchers should you listen to? Here are three suggestions: John Anderson, David Geary, and Robert Siegler -- three highly distinguished scientists in the psychology of math education. What are they likely to tell you? I believe you will get strong agreement from them on the following points: **that varied and repeated practice leading to rapid recall and automaticity is necessary to higher-order problem-solving skills in both mathematics and the sciences.**
- They would probably explain to you that lack of automaticity places limits on the mind's channel capacity for higher-order problem-solving skills. They would tell you that **only intelligently directed and repeated practice, leading to fast, automatic recall of math facts, and facility in computation and algebraic manipulation can one lead to effective real-world problem solving.** Anderson, Geary, and Siegler would provide you with reliable facts, figures, and documentation to support their position, and these data would come not just from isolated lab experiments, but also from large-scale classroom results. If these top scientists agreed on all these points, that is the consensus you should trust, **no matter how many pronouncements to the contrary might be made by national educational bodies.**

Jo Boaler had never memorized her times tables -- “It has never held me back, even though I work with maths every day.” That is only possible because she works with Fuzzy Math every day.

Times Table *Torment*?

Even now, the fissure between traditionalists and progressives can erupt. Boaler has plenty of skeptics who think her ideas sacrifice rigor. In 2015, she ignited controversy in Britain by saying at a conference **that she had never memorized her times tables. “It has never held me back, even though I work with maths every day,”** she said. “It is not terrible to remember math facts; what is terrible is sending kids away to memorize them and giving them tests on them, which will set up this maths anxiety.”

Charlie Stripp, director of England’s National Centre for Excellence in the Teaching of Mathematics, struck back in an op-ed in the education publication *Tes*. **“It is not the learning of times tables that is causing anxiety but rather it is lack of times table knowledge,”** he wrote. **“It should be an educational entitlement that all children are helped to learn their times tables.”**

“I can multiply”: Local group shows importance of knowing multiplication



Travis Bolden <https://www.icanmultiply.org/>

The Northridge Chronicles: *A Virtual Play*

- If an 11th or 12th grader was solid on arithmetic, algebra and trigonometry then the Calculator may be of some use. But to use the calculator to teach trigonometry and graphing has been a failure. **I have 90 students this semester and most used calculators in high school and/or pre calculus. I don't have one student who has gotten a grade higher than a C this semester.** Dave Protas told me his averages in 103 have been around 40% (and he is an acknowledged excellent teacher). When I first started teaching here my mean grade in Calculus was 20 points higher and those students were not educated with calculators.

What is Changing in Math Education? by Mathematically Correct

- There is grave danger that calculators, used too early, can **seriously impair** the student's ability to perform and understand simple calculations. The need for this understanding is not obviated by the availability of calculators and computers.
- Yet another point emphasized by these new programs is the use of calculators and computers. Based on the view that we live in an increasingly technological society, these programs introduce the use of calculators as early as kindergarten, and usually require students to have them available at all times. **The idea is that students shouldn't have to be bogged down with mundane things like addition and subtraction, since calculators can do these things for them. At higher levels, calculators that do fraction problems or graphs are required.** Opponents argue that **the use of calculators in the new programs is excessive and leads to a deficit of basic skills.** Algebra students have been known to reach for a calculator when faced with the multiplication of two single-digit numbers or needing to divide 300 by 3.

Section E

Knowledge and Skills are Fundamental;

Memorization is Essential;

“Critical Thinking,” “Conceptual Understanding” Without Foundational Knowledge Leads Only to “Rote Understanding.”

CMF 2021:

"The wistful or nostalgic 'back-to-basics' approach that characterizes the Board standards overlook the fact that the approach **has chronically and dismally failed**. It has excluded youngsters from engaging in **genuine mathematical thinking and therefore true mathematical learning**, and has proposed a disproportionate mathematically illiterate citizenry."

"...school systems that embark on a course that **substitutes computational proficiencies for a commitment to deep, balanced, mathematical learning**."



Why do students fail calculus? by Stanley Ocken

- students will use calculus as an integral part of their professional activities. Nearly all will use the analytic and algebraic skills honed in calculus courses, and **it is a lack of algebraic skills that poses the principal obstacle to students' success in calculus.**
- It is borne out by long experience that students with flawless algebra skills usually do well in calculus, while those with even moderate weaknesses in algebra perform poorly. In this respect, mathematics is unique. An essay with a few spelling and grammar mistakes is still intelligible. A couple of missed notes in the performance

of a piano sonata don't make a difference. **In mathematics, however, there is much less room for error.** If a student makes an algebra mistake at the beginning of a problem, the remainder of the solution may be rendered completely irrelevant to the stated problem. **The plight of a student doesn't notice that the algebra has gone astray is analogous to that of a deaf pianist who doesn't notice her hands slipping laterally a key or two.** Cacophony ensues.

that Isaac Barrow, Newton's professor, has prior claim on many of the foundational concepts of the subject. However, Barrow's name is obscured from history books because he failed to develop mathematical notation that would facilitate working with those foundational ideas and applying them to the solution of real world problems. In short, **Barrow knew the concepts, but he didn't have the right symbols and algebraic methods to work with.**

How Not to Teach Math by Matthew Clavel

- Defenders of critical thinking say we need to rescue our schools from a repressive “drill-and-kill” pedagogy that makes children automatons, spitting back the facts and rules that teachers have drummed into their heads and never learning to think on their own. The truth, of course, is that no one claims that knowing how to think independently isn’t important. But **thinking can’t take flight unless you do know some basic facts**—and nowhere is this more the case than in math. **If you really want your students to engage in “higher-order thinking” in math, get them to master basic operations like their times tables first.** When a middle schooler is learning to factor equations in eighth grade, it’s a crippling waste of mental energy if he needs to figure out how many times four goes into 20. Mastering fundamentals through practice can lift a child’s confidence to do harder work.
- Unfortunately, a student in a Fuzzy Math program—including *Everyday Mathematics*—is unlikely to master much of anything. The hours of logically linked lessons that old-style math classes spent on practicing operations so that they became second nature to students just are not there.
- **Instead of rote learning and memorization, students move haphazardly from one seemingly unconnected topic to another. In Fuzzy Math lingo, it’s called “spiraling.”** On this view, teachers shouldn’t use a single method to get addition across to students; they should **try lots of approaches—like adding the left-most digits first.** That way, the Fuzzy Math approach says, you have a better chance of getting students to understand the concept of addition. **In practice, however, trying to teach a host of different methods if students haven’t sufficiently mastered any specific one—as is all but inevitable, since they haven’t spent much time *practicing* any specific one—can be very confusing.**
- Equally mystifying, *Everyday Mathematics*, like Fuzzy Math programs generally, **abruptly introduces concepts like basic algebra that students aren’t officially taught until years later.**

- [The Difference Between Thinking and Knowing: Memorization Doesn't Deserve Its Bad Name](#) by Claudia Winkler

To commit something to memory isn't necessarily to learn it 'without understanding or thought.' As anyone knows who's tried it, retaining facts is much easier when you see how they fit into a larger picture that makes sense. Yet in a subtle bit of linguistic sleight of hand, the pejorative term 'rote memorization' is commonly used as synonymous with memorization tout court. It's almost always contrasted with comprehension and critical thinking--as if knowing things and thinking about things were mutually exclusive. ... One can't help wondering what it is the children are to analyze -- what exactly they are to think about -- if their starting point is not to be a command of the specifics recounted in the book. **This conflation of mindless, blab-school, learning-by-rote with the necessary, if sometimes painful, committing of information to memory has a sordid effect: to dress up ignorance as superior thoughtfulness.** Implicitly, **it disparages the intake of knowledge** -- once the very essence of classroom learning -- as an activity fit only for drones.

- [Why do students fail calculus?](#) by Stanley Ocken

Received doctrine that permeates the literature and practice of K-12 mathematics education, if implemented uncritically, will obstruct the development of skills that are critical for the study of calculus. The following related **false dichotomies** are addressed in this paper:

Blind rote is always bad and leads to error, whereas thinking about what you are doing is always good and leads to success.

Solving problems by using algebra is bad, whereas concretizing problems by using **physical and visual models** is good.

Memorization is a component of critical thinking, not its opposite.

by Erica Meltzer



- ...ideas about education have become polarized: on one side, joyless, dry, rote learning, devoid of imagination or interest, with no other end than the thoughtless regurgitation of facts; on the other side, a sort of kumbaya, free-to-be-you-and-me utopia, where learning is always an imaginative and exciting process with no wrong answers or unpleasantness.
- Real education is most certainly not about learning to fill in little bubbles, and at its best, it can be wonderful and stimulating and engaging. But *can be wonderful and stimulating is not the same thing as must never be boring or involve any sort of protracted struggle*, and there seems to be a camp that conflates the two. *Some things are hard; that's called life*. As someone who spends a lot of time teaching students fundamentals that they haven't acquired in school, I find just as disturbing — and, frankly, bizarre — *the idea that those "boring" fundamentals can simply be bypassed in favor of "higher level critical thinking skills."* Yet that idea seems to be have taken root rather tenaciously.
- On the other hand, there is also a type of education that views rote learning as a means to an end — one that recognizes that factual knowledge is actually the basis for higher level thinking. This type of rote knowledge is also known as "inflexible knowledge." Cognitive scientist Daniel Willingham has written extensively *about the problem with treating critical thinking as something that can be taught in the abstract*.
- ... you don't acquire higher-level skills without mastering lower-level skills first. If you skip over the fundamentals, you might stagger along looking like you know what you're doing for a while, but sooner or later, you're going to crash.

Memorization is a component of critical thinking, not its opposite by Erica Meltzer

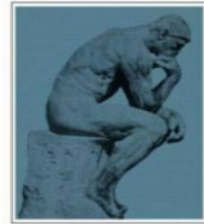
- Willingham's research flies in the face of much of the educational status quo. One belief currently rampant is that students no longer need to memorize factual information because technology has made that information available to them at the click of a button. Because they no longer have to "waste" brainpower memorizing, or so the line of thinking goes, their minds will be freed up for "higher level critical thinking." The problem with this view is that it overlooks the fact **that critical thinking emerges from the scaffolding provided by rote knowledge; it can't be divorced from it.** When you know facts and dates and concepts by heart, it becomes much easier to see the relationships between them. It doesn't mean that you'll automatically see the relationships between them (that's the point of education), but you will have a stronger basis for doing so.
- I don't think most Americans would argue with that idea when it comes to, say, sports or music. They would consider it basic common sense that **top athletes don't simply jump into a high level of competition after a little bit of haphazard training. If they're not ready, they'll get injured badly.** Likewise, a musician who hasn't mastered basic scales isn't usually encouraged to schedule her solo debut. It's understood that **years of practice and repetition are required, some of which is "fun" and much of which is not, and that fundamental skills must be mastered before more advanced ones are introduced.**

Memorization is a component of critical thinking, not its opposite by Erica Meltzer

- Yet that is more or less the equivalent of what an awful lot of people seem to expect high school student to be able to accomplish academically. Not only is it unrealistic to ask high school students to write papers showing evidence of complex, critical, “high-level” thinking without giving them the grammatical, rhetorical, analytical, literary, historical, and cultural knowledge (among other things) to actually perform that kind of analysis, but it’s downright delusional.
- Listening to [ed-school grads](#) rhapsodize about the joys of learning, you have to wonder whether they’ve actually ever seen students at home, sobbing hysterically and making their parents nuts as they try to eek out a couple of semi-coherent paragraphs.
- So if teachers aren’t teaching the basics well or in a manner that engages students’ interest, **it simply means that those basics need to be taught better — not that they can or should be discarded as irrelevant.** A terrible teacher can massacre even the most fascinating subject, **and an exceptional teacher can teach the basics clearly and directly in a highly engaging manner.** From what I’ve seen, **students are incredibly grateful when the latter occurs. They end up with a real sense of accomplishment rather than the feeling that they’re grasping a straws.**
- [administrators] insist that teachers facilitate high level critical thinking **while simultaneously discouraging them from reinforcing the kind of fundamentals that are necessary for critical thinking to occur. It’s positively schizophrenic.**

Thinking Tends to Focus on a Problem's "Surface Structure"

With Deep Knowledge, Thinking Can Penetrate Beyond Surface Structure



COGNITION
The Thinking Animal
THIRD EDITION

DANIEL T. WILLINGHAM



WHICH LEARNING
STRATEGIES DO & DON'T
WORK

with Dr. Daniel T. Willingham



Knowing that one should think critically is not the same as being able to do so. That requires domain knowledge and practice.

Daniel T. Willingham:

People who have sought to teach critical thinking have assumed that it is a skill, like riding a bicycle, and that, like other skills, once you learn it, you can apply it in any situation. **Research from cognitive science shows that thinking is not that sort of skill. The processes of thinking are intertwined with the content of thought (that is, domain knowledge).** Thus, if you remind a student to “look at an issue from multiple perspectives” often enough, he will learn that he ought to do so, but if he doesn’t know much about an issue, he can’t think about it from multiple perspectives. You can teach students maxims about how they ought to think, but without background knowledge and practice, they probably will not be able to implement the advice they memorize. Just as it makes no sense to try to teach factual content without giving students opportunities to practice using it, **it also makes no sense to try to teach critical thinking devoid of factual content.**



Facts Are Fun. So Why Do Educators Hate Them? by Bruce Price

Anyone who has watched Jay Leno go 'Jaywalking' knows that many adults today, even ones who attended college, are remarkably unacquainted with even rudimentary knowledge.

One week he asked this question: 'What body of water lies to the west of California?' Remember, the show is shot in California. But he found people who did not know! ...

One of the weirdest tenets of modern educational theory is that children can engage in 'critical thinking' without knowing any facts to think about. **Sort of like playing tennis without a ball, swimming without water, or conducting chemistry experiments without chemicals. These activities are properly called make-believe. Common sense says that students should first learn facts, then analyze those facts.** We see lots of surveys in the media indicating that Americans can hardly find their own state on a map. They don't know where Iraq and Vietnam are. They also don't know the simplest kind of information, such as: Roughly how much of the Earth is covered by water--30%, 50%, 70%? The tallest mountain on this planet is what? How many quarts in a gallon? Call me old-fashioned but I think everyone has to know this basic stuff.

Section F

Manipulatives, Visuals, Finger Math, Multiple Representations, and Brain-based Learning: These Reform Math Darlings May Do More Harm than Good

CMF 2021: Researchers Joonkoo Park and Elizabeth Brannon reported that different areas of the brain were involved when people worked with **symbols**, such as numerals, than when they worked with **visual and spatial information**, such as an array of dots. The researchers also found that mathematics learning and performance were optimized when these two areas of the brain were communicating with each other. **Learning mathematical ideas comes not only through numbers, but also through words, visuals, models, algorithms, multiple representations, tables, and graphs; from moving and touching; and from other representations.** But when learning reflects the use of two or more of these means and the different areas of the brain responsible for each communicate with each other, the learning experience improves.

CMF 2021:

- Human minds want to see and understand patterns. But the joy and fascination young children experience with mathematics is quickly replaced by dread and dislike when mathematics is introduced as a dry set of methods they think they just have to accept and remember.
- While instruction has shifted toward learning with understanding, and the field increasingly attends to issues of equity and access, mathematics education still largely recreates this **rigid and rote approach** to mathematics teaching and learning; achievement in mathematics often reflects these original, narrow purposes. These foundations continue to limit the experience of mathematics as relevant, meaningful, and engaging and obscure many student competencies that could otherwise be drawn upon to support **making sense of mathematics**. This is particularly true for linguistically and culturally diverse learners of English, whose competencies have long been obscured through deficit frameworks and **narrow conceptions of mathematical competence**.
- As research has consistently pointed out, **calculator use does not hinder the learning of rich mathematics**. It does hinder the learning of procedural mathematics, however, especially when that is believed to be the primary objective. In considering the use of technology, **the belief that rote algorithms and procedural skills (which are easily replaced by calculators) are the most important mathematics to be learned which must be reconsidered**.



Don't Be Manipulated, Center for Education Reform

The use of "manipulatives" has become a buzzword in education circles. The concept refers to kindergarten and elementary students' use of concrete objects - anything from blocks or magnetic letters to complete systems specially designed for use in the classroom - for, literally, hands-on learning of math and language concepts.

However, a recent study brings into question their effectiveness - in this specific case, the use of a particular set of manipulatives usually did not transfer into faster or greater proficiency in the symbolic, written worlds of math and language.

... whether their effectiveness has ever been independently assessed. And finally, make sure your school keeps the focus on results, and not just process.

Manipulatives must be used with discrimination, for reliance thereupon can foster habits of mind that damage students' mathematical development.

What to do about Canada's declining math scores, by Anna Stokke

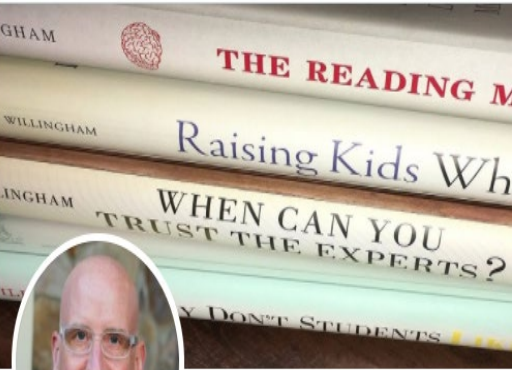
Overemphasis on hands-on materials and pictures also presents problems. Although some materials, such as base-ten blocks, might assist initial learning, overuse can prevent the transfer of information to long-term memory, because working memory is assaulted with extraneous information. Transfer is more likely to occur if mathematical symbols are stressed over concrete materials (Kaminski, Sloutsky, and Heckler 2009).

Mindful of Symbols: Educational Ramifications by Judy S. DeLoache

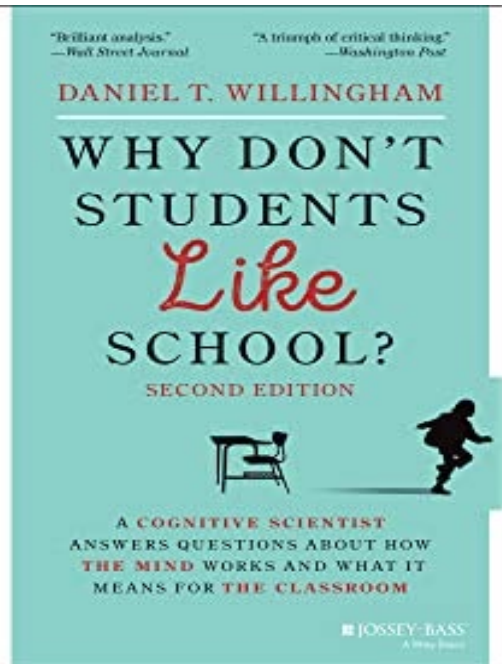
- **Teachers in preschool and elementary school classrooms around the world use 'manipulatives' -- blocks, rods and other objects designed to represent numerical quantity. The idea is that these concrete objects help children appreciate abstract mathematical principles. But if children do not understand the relation between the objects and what they represent, the use of manipulatives could be counterproductive. And some research does suggest that children often have problems understanding and using manipulatives.**

Meredith Amaya of Northwestern University: Uttal and I are now testing the effect of experience with symbolic objects on young children's learning about letters and numbers. Using blocks designed to help teach math to young children, we taught six- and seven-year-olds to do subtraction problems that require borrowing (a form of problem that often gives young children difficulty). We taught a comparison group to do the same but using pencil and paper. **Both groups learned to solve the problems equally well--but the group using the blocks took three times as long to do so. A girl who used the blocks offered us some advice after the study: 'Have you ever thought of teaching kids to do these with paper and pencil? It's a lot easier.'**

Do Visual, Auditory, and Kinesthetic Learners Need Visual, Auditory, and Kinesthetic Instruction?



Daniel Willingham



The idea that people may differ in their ability to learn new material depending on its modality -- that is, whether the child hears it, sees it, or touches it -- has been tested for over 100 years. And the idea that these differences might prove useful in the classroom has been around for at least 40 years. **What cognitive science has taught us is that children do differ in their abilities with different modalities, but teaching the child in his best modality doesn't affect his educational achievement. What does matter is whether the child is taught in the content's best modality. All students learn more when content drives the choice of modality.** In this column, I will describe some of the research on matching modality strength to the modality of instruction. I will also address **why the idea of tailoring instruction to a student's best modality is so enduring -- despite substantial evidence that it is wrong.**

How Has Modality Theory Been Tested? The Content's Best Modality Is Key

The research presented in this article boils down to this: Modality of instruction is important, but it is equally important for all students -- not more or less important depending on students' modality preference. There are several important implications for educators. First, **teachers need not worry about differences between students in terms of modalities; there are not visual or auditory or kinesthetic learners. Indeed, applying this incorrect theory may actually shortchange some students.**

What is Changing in Math Education? by Mathematically Correct

- Another philosophical notion is the idea of *Complete Math*, which is a replacement term for *Whole Math*. Just as *Whole Language* attempted to skip the basics of phonics and go directly to reading literature, *Whole Math attempts to cast aside computational basics and go to final productions that rely on math at some level. This view holds that math is not used only in equations, but in writing and discussion as well. The implication is that students should write essays and have group discussions about math.* The major problem with this method is that students end up spending hours working on essays, again **detracting from their chance to practice basic skills**. This has the substantial risk that potentially controversial moral lessons make their way into assignments.
- Proponents of *Complete Math* like to talk about "**communicating mathematically**." However, the students do not learn the language of mathematical exposition -- the terminology, symbols, and syntax needed for communicating mathematically. Instead, their products are better characterized as "**communicating about math**," -- written and spoken words and pictures that have something to do with math but are a far cry from "communicating mathematically." The process leaves many wondering what English-math or art-math is.

- Let's start the discussion of "brain-based learning" with [an in-depth review](#) of one of the more widely distributed books on this, [Teaching With The Brain In Mind](#), by Eric Jensen.

It's attractive designed, sure enough, but the content is so grossly flawed it's funny. **This book (and others like it) are used at many progressivist schools to justify dubious and dangerous curriculum changes.**

- [Buyer Beware: Too Early To Use Brain-Based Strategies](#) by Kathleen Madigan, Ed.D., Executive Director of the National Council for Teacher Quality

Why is this a symbol of what's wrong with education? What is not wrong is wanting to help all students learn and trying to find methods to do so that are supported by research. What is wrong is that, once again, **educators have taken a leap of faith rather than use good science, impeding the development of a professional knowledge base.**

The trouble is that some educators are extrapolating piecemeal from certain findings and creating curriculum specifications without actual research to back up their claims. They are hitting the streets with 'brain-based' learning kits and workshops. ... **Using the term 'brain-based' has become fashionable, but unfortunately, it is only that -- a fashionable fad that may actually undermine serious research in a very complex field.**

In fact, actual testing of brain-based theories in classrooms is almost non-existent.

Hard Lessons – *The Atlantic's* Interview with Diane Ravitch, 2000

Another troubling trend, which seems to be emergent, is a great fascination with what's called "**brain-based**" learning. This is apparently a distortion of what cognitive scientists have learned about how children learn. There are scores of workshops being offered now to teachers and administrators on **brain-based assessment, brain-based learning, and brain-based supervision**. But the most advanced cognitive scientists today say that we don't know nearly enough about studies of the brain to be able to draw practical implications for the classroom. I think that this is yet another trend that may seem to offer easy answers to learning, instead of getting down to the business of preparing excellent teachers of mathematics and science and history and literature and foreign languages and the arts who can teach students what they need to learn during the time they're in school.

Each To Their Own: The [British] government espouses the theory of learning styles with scant regard to the evidence, by Phil Revell

Howard Gardner ... never intended his book on multiple intelligences (MI) to be a blueprint for learning, but he was aware that many educationalists were adapting his ideas. The shock came on a visit to Australia. 'I learned that an entire state had adapted an education program based in part on MI theory,' he says. 'The more I learned about this program, the less comfortable I was. **Much of it was a mishmash of practices -- left brain and right brain contrasts, sensory learning styles, neurolinguistic programming and multiple intelligences approaches, all mixed with dazzling promiscuity.**' Gardner says he is still 'uneasy' about the way his theories are used in schools. But other researchers are less picky; there is a range of consultants willing to accept large fees from schools. Few mention the scientific doubts about the approach they are selling.

From: <http://www.illinoisloop.org/brain.html>

- These issues contain a variety of articles -- articles by advocates of brain-based curricula, articles by educational futurists, articles by cognitive (not brain) scientists. In fact, **it is rare to find an article written by a neuroscientist in the educational literature.** Of these articles, those citing cognitive research ... provide the most useful advice to educators. Educators should be aware that cognitive science - the behavioral science of the mind - is not the same as neuroscience - the biological science of the brain. ... **Most other claims found in the emerging brain and education literatures are vague, outdate, metaphorical, or based on misconceptions.**
- **...Despite all the interest and media attention, we currently do not know enough about brain development and neural function to link that understanding, in any meaningful way, to educational practice.**
- **...We should be wary of claims that neuroscience has much to tell us about educational practice.**
- **...Neuroscience has discovered a great deal about neurons and synapses, but not nearly enough to guide educational practice in any meaningful way.**



- "...Scientists must ... undertake a good faith effort to make sure that the fruits of science are applied wisely and not foolishly. ... Let me introduce an example from my own work as a cognitive psychologist. Nearly twenty years ago, I developed a new theory of intelligence called the theory of multiple intelligences. **While I thought that this theory would be of interest primarily to other psychologists, I soon discovered that it was of considerable interest to educators all over the world. Educators began to make all kinds of applications of the theory.** I was intrigued and flattered by this interest. ...
- "About ten years later, I received a message from a colleague in Australia. He said, 'Your multiple intelligences ideas are being used in Australia and you won't like the way that they are being used.' I asked him to send me the materials and he did so. My colleague was absolutely correct. The more that I read these materials, the less I liked them. **The 'smoking gun' was a sheet of paper on which each of the ethnic and racial groups in Australia was listed, together with an explicit list of the intelligences in which a putatively strong and an accompanying list of intelligences in which they were putatively weak.**
- "This stereotyping represented a complete perversion of what I personally believed in. If I did not speak up, who would? Who should? And so, I went on television in Australia and criticized the program as pseudo-science. That critique, along with others, sufficed to result in the cancellation of the project."

-- Howard Gardner

- The truth is that **there is virtually no support for learning styles** in the research literature. While students may have preferences, all of us (with very rare exceptions) learn by seeing, hearing, and doing. Likewise, all of us (with very rare exceptions) think verbally, mathematically, and spatially. So teachers should be trying to provide students with the content knowledge, experiences, and skills that support development of all three ways of thinking. ... Instead of tailoring lessons to students' supposed learning styles, teachers should be concerned with tailoring their lessons to the content (e.g., showing pictures when studying art and reading aloud when studying poetry).

-- Nora S. Newcombe, American Psychology Association

- ...a short, common-sense look at the Multiple Intelligences fad that concludes, “**Too much attention to ‘intelligences’ can cause a teacher, and even an entire school, to digress into fun and interesting activities that do not promote real academic achievement.**”

-- Eric Buehrer

Seven Kinds Of Smart, by James Collins

- As science, then, there may be less to the theory of multiple intelligences than many educators seem to believe. ... evidence for the specifics of Gardner theory is weak, and there is no firm research showing that its practical applications have been effective. ... **The danger is that it leads to wasted time, to an emphasis on less important skills and to a false sense that learning has taken place when it has not.** ... 'The discussion is all hunch and opinion,' wrote George Miller, one of the founders of cognitive psychology. ... The most common use of MI is to attack a topic from seven directions to fit in all the intelligences. ... **All these activities will take up a lot of time, and they will teach children very little ...**

From <http://www.illinoisloop.org/mi.html>:

- Virtually all of the fountains of rhetoric that pour from Multiple Intelligences (MI) theory **espouse use of MI "intelligences" to design various constructivist child-centered projects.** Thus, MI is welcomed **as a way to bolster support for progressivist methods**, rather than adding much new. Almost never is there discussion of alternative *teacher*-centered methods. For example, a truly effective teacher might make use of a few classroom demonstrations, a video clip, interactive choral response, and an out-loud story, in addition to or in place of straightforward lectures. In fact, an effective teacher might well accomplish *all* of this, and with greater learning effectiveness, in the same amount of classroom time as a single one in a suite of constructivist "projects" used in the name of MI.

Section G

Open-Ended Questions,
Open Tasks, Projects,
and Investigations Leave
the Chance for Failure
Wide Open.

CMF 2021:

...we need to **change classroom approaches from work on short questions to instruction that engages students in rich, deep tasks that honor students' ideas and thinking and draws on their cultural backgrounds as resources.**

One of the reasons that students are often limited when in tracked groups is the questions given to the students are **narrow, which precludes access for some students** and stops higher achievers from taking the work to higher levels.

CMF2021:

The principle derives from a need to confront the mile-wide but inch-deep mathematics curriculum experienced by many. This framework's answer to the coverage-versus-depth challenge posed by the principle of focus is to lay out principles for instructional design that make the Standards achievable, including: (a) focus on big ideas; (b) use **tasks** worthy of student engagement; and (c) embed exercises in a larger context of **investigation**.

Reform Mathematics Education: *How to "Succeed" Without Really Trying*

by Paul Clopton

- **Projects** - The reform programs are loaded with projects and activities, often called **investigations**. Part of the argument for these methods relates to stimulating student interest. There are also claims of richer mathematics and the importance of context. Even a casual inspection of these activities will show that **they tend to be very time consuming while involving very little mathematics**. Time for mathematics, both in class and at home, is seriously limited and must be used as efficiently as possible. These activities are inefficient learning methods. But, **beyond that limitation, they promote the evaluation of students on the basis of non-mathematical dimensions such as how artistic the display is or the writing style of the report or the social value of the application.**

The Mathematics Program Advisory of 1996

- There are good reasons to be concerned about the direction of mathematics education in California and the rest of the United States. The members of Mathematically Correct are particularly opposed to various *reform* programs designed to be in agreement with either the Standards of the National Council of Teachers of Mathematics or the 1992 California Mathematics Framework. **These reforms engender low expectations, the de-emphasis of basic skills, and general disregard for standardized tests as measures of student achievement. Such changes threaten to leave our children unprepared for the demands of the modern world.** The California Mathematics Framework has guided the purchase of perhaps hundreds of millions of dollars worth of textbooks and math education materials that place **greater emphasis on working in groups, writing about math and feeling good than they do on learning and mastering the basic skills and getting the correct answers to problems.**
- The Program Advisory notes that basic skills **should be used routinely and automatically, should be practiced regularly, and many should be committed to memory.** The document makes it clear that many aspects of mathematics, above and beyond addition and multiplication, need to become automatic and routine. This allows basic skills at all levels to become goals in and of themselves, and encourages students to practice these skills directly. **No longer is it necessary to address basic skills only within the context of open-ended problems as is common in the Framework-aligned texts.**
- The Program Advisory makes it clear that one of the goals of problem solving is to allow children to practice various kinds of problems until they become routine. It also stresses that students need computation skills, skills related to solving equations or inequalities, graphing skills, knowledge of geometric relationships and measurement concepts, and the analysis of graphical information and data. A mathematics program should include a substantial number of *ready-to-solve* exercises that are designed specifically to develop and reinforce these basic technical skills. **These statements are critical as they indicate that a balanced mathematics program can and should include use of repeated exercises representing the kinds of problems that recur in a number of different situations, and that a good education in problem solving is not achieved by programs in which problem solving is taught exclusively via a small number of perplexing or open-ended problems.**



Cognitive Child Abuse in Our Math Classrooms, by C. Bradley Thompson

- The test results are in: America's children are flunking math. ... As educators scramble to explain America's math meltdown ... few are willing to look at the fundamental cause: the new, 'whole-math' method for teaching ...
- In a typical whole-math classroom, children do multiplication not by learning the abstract multiplication table, but by using piles of marshmallows. They count a million birdseeds in order to understand the concept 'million.' They measure angles by stretching rubber bands across pegged boards.
- One whole-math program preposterously claims to foster a 'conceptual understanding' of math by asking fifth-graders the following stumper: 'If math were a color, it would be _____, because _____.' **Surely such exercises foster in children only conceptual stultification -- along with a bewildered sense of frustration and disgust.**
- Another whole-math program asks sixth-graders to address the following problem: 'I've just checked out a library book that is 1,344 pages long! The book is due in three weeks. How many pages will I need to read a day to finish the book in time?' The proper way to solve the problem would be to use the method for long division: 1,344 divided by 21. **By contrast, the whole-math approach assigns students to a group, requires them to design their own problem-solving rules, and urges them to guess if all else fails. In other words, children are told that their random 'strategies' are just as good as the logically proven principles of long division. They are taught that the vote of the group, rather than the reasoning of the individual mind, is the means of arriving at the truth.**
- Now imagine flying on a plane designed by aeronautical engineers who have been trained to concoct their own math schemes and to use a 'guess-and-check' method.

- [The "Laws" of Learning](#) by Laurie H. Rogers

"A central tenet of ... **constructivist teaching** is that children should work **cooperatively** in groups to 'explore' and 'discover' ... figure out concepts on their own. Reformers say this method makes [school] interesting and fun and leads to 'deeper understanding.' ... I'm not sure how much fun this process actually is for the students, who tend to be concrete thinkers and who generally appreciate straightforward, logical approaches to learning. **Experimentation in groups can be fun for them, but I suspect they'd rather it come in small doses. Otherwise, they can become stressed out trying to teach themselves 5,000 years of math in the small snippets of time they have available to them.**

- **Angry Parents, Failing Schools**, by Elaine McEwan

In a fever to "reform" our schools for "21st century" "higher-order thinking skills", educational bureaucrats have reduced the amount of substantive content in curricula and replaced it with **touchy-feelie psychobabble encouraging feelings, writing about problems instead of solving them, and meaningless goals**. McEwan, an award-winning school principal takes the educrats to task. Warning: the subject is not inner-city schools; it may very well include the school YOUR kids attend. Yes, YOUR kids!



Sally says students need more than math ‘appreciation’

- Paul Sally Jr. says many U.S. college-level mathematics courses for nonmajors are “the equivalent of people taking English and reading classic comics.”
- Sally, Professor in Mathematics and the College, delivered this message during an invited address Saturday, Jan. 18, at the Joint Mathematics Meetings of the American Mathematical Society and the Mathematical Association of America.
- **The trend toward teaching college students about mathematics rather than teaching them to actually do mathematics has led to growing numbers of college graduates who are numerically illiterate**, Sally said. “There is almost no situation in life where you’re not going to need some quantitative literacy in order to achieve certain goals,” he said.

“This notion that one has to ‘interest’ students in mathematics in order to make them do it has gone much too far, to the point where **real mathematics in many cases has just disappeared entirely from the courses**. They’re just a discussion of what mathematics does and beautiful pictures and imprecise ideas.”

But the trend nationwide is toward less rigorous courses. As evidence, he points to a textbook originally published in the 1940s called *What is Mathematics?* Endorsed by Albert Einstein, it was written to explain mathematics to nonmathematicians. “It is now regarded as much too hard for that purpose,” said Sally. In his talk, Sally presented examples of problems that lend themselves to quality instruction. “**You could start at very early levels, third or fourth grade, and build on them in such a way that they become serious problems even at the graduate research level. You just have to bring them along,**” Sally said.

-- Paul Sally Jr., professor in mathematics, University of Chicago, the “math pirate”

Section H

Reflecting the Progressive Era Heritage of Preparing Students for Low-skill Jobs and Practical Living, the “Authentic,” “Real-world,” “Relevant,” “Problem-Solving” Reform Math Take Students Away From Abstraction and Rigor.

CMF 2021:

- **Real-world tasks** can offer students opportunities to mathematize contexts that connect to their lived experiences.
- An **authentic problem, activity, or context** is one in which students investigate or struggle with situations or questions about which they actually wonder. ... an **activity is inauthentic if students recognize it as a straightforward practice of recently-learned techniques or procedures**, including the repackaging of standard exercises in forced “real-world” contexts. . Mathematical patterns and puzzles can be more authentic than such real-world settings.
- **Data Science tasks**, such as those we highlight in the Data Science chapter and in the chapter for grades 6–8, **are naturally open**, and provide many opportunities for students **to connect mathematics to their lives**. Students can, for example, design wheelchair ramps, plan a new school garden, or survey peers to find out how they have been impacted by distance learning, drawing from their own knowledge and interests as they learn new mathematics. These tasks that draw from students’ lives are very different from the imagined contexts that often fill textbooks and present mathematics in ways that students may find **irrelevant and “other worldly.”**
- Current structures often reinforce existing factors that allow access for some while telling others they don’t belong; structures must instead challenge those factors by **providing relevant, authentic** mathematical experiences that make it clear to all students that mathematics is a powerful tool for making sense of and affecting their worlds. This will be an important contribution to equitable outcomes.

CMF 2021:

This definition expresses the basis of mathematical rigor: reasoning which enables understanding “all the way down to the bottom,” often expressed in terms of validity and soundness of arguments. According to the definition used here, conceptual understanding cannot be considered rigorous if it cannot be *used* to analyze a novel situation encountered in the world; computational speed and accuracy cannot be called rigorous unless it is accompanied by conceptual understanding of the strategy being used, including why it is appropriate in a given situation; and a correct answer to an application problem is not rigorous if the solver cannot explain to the client both the ideas of the model used and the methods of calculation.

In particular, **rigor is *not* about abstraction**. In fact, a push for premature abstraction leads, for many students, to an absence of rigor in the sense used in this framework. It is true that more advanced mathematics often occurs in more abstract contexts. This leads many to value more abstract subject matter as a marker of rigor. “**Abstraction**” in this case usually means “less connected to reality.”

International Tests are Not All the Same, by Tom Loveless



- The PISA math assessment is based on a philosophy known as **Real Mathematics Education (RME)**, championed by the Freudenthal Institute in the Netherlands. Jan de Lange of the Freudenthal Institute chairs the [PISA expert group](#) in mathematics. **RME's constructivist, problem solving orientation is controversial among mathematicians.** In the U.S. in the 1990s, a coalition of mathematicians, parents, and local educators opposed similar types of curricula in what became known as the “math wars.”

TIMSS and Program for International Student Assessment (PISA) are quite different. TIMSS is **curriculum-based, reflecting the skills and knowledge taught in schools**. PISA assesses whether students can apply what they've learned to **solve “real world” problems**.

The contrast is stark between **constructivist countries** and **those favoring more traditional math curricula**. On PISA, New Zealand scores within 27 points of Korea (519 vs. 546). On TIMSS, New Zealand and Korea are separated by a whopping 125 points (488 vs. 613), a difference of nearly one full standard deviation between the two tests! Chinese Taipei outscores Finland by only 2 points on PISA (543 vs. 541, the scores are statistically indistinguishable)—but by 95 points on TIMSS (609 vs. 514).

Please remember that PISA is **not a curriculum-based test**, and whether we want schools to teach a mathematics curriculum compatible with PISA is a proposition that many people find **objectionable**.

International Mathematics Achievement (Average Scale Scores)



**East Asian Countries Top Achievers in
Mathematics by a Substantial Margin**

Singapore **616** • Chinese Taipei **612** • Korea, Rep. of **607**

Japan **594**

Hong Kong SAR **578**

Russian Federation **543** • Ireland **524** • Lithuania **520** • Israel **519**

Australia **517** • Hungary **517** • United States **515** • England **515**

Finland **509** • Norway (9) **503** • Sweden **503** • Cyprus **501** • Portugal **500**

Italy **497** • Turkey **496** • Kazakhstan **488** • France **483** • New Zealand **482**

Bahrain **481** • Romania **479** • United Arab Emirates **473** • Georgia **461**

Malaysia **461** • Iran, Islamic Rep. of **446** • Qatar **443** • Chile **441**

Lebanon **429** • Jordan **420** • Egypt **413** • Oman **411** • Kuwait **403**

Saudi Arabia **394** • South Africa (9) **389** • Morocco **388**

International Tests are Not All the Same, by Tom Loveless

Finland is a great country and makes a wonderful travel destination. It also has fine schools. But its reputation in education is a bit overblown, based primarily on high PISA scores and an aggressive educational tourism industry. **Like New Zealand, Finland also has a math curriculum compatible with PISA.**

The U.S. press has not told the entire story about Finland. Not everyone in Finland has applauded the emphasis on real world math. **In 2005, a petition signed by more than 200 university mathematicians complained that, despite the country's high PISA scores, students were increasingly showing up for college unprepared in mathematics.** An analysis of items on a Finnish matriculation exam **revealed a sharp fall off in computation skills**, particularly with problems involving fractions and exponents. The concern in Finland about the math curriculum mirrors that expressed in the 1990s about math reform in the U.S.

2. Mathematics curriculum—changes and effects

The changes in the mathematics curriculum in Finland have followed the international trends. Since 1970 three major revisions have taken place. The first was influenced by the so-called New Math. This created a lot of discussion but had a relatively small effect. The second revision can be labelled “Back to basics”. The last change “Problem solving” had a much greater impact. It was very much influenced by the demand that the applications of mathematics are all important—mathematics as such has little value. The influence of calculators was also profound. It was thought unnecessary to teach those skills which can be performed by a calculator. Similar changes were experienced in other OECD countries.

In Finland these trends had the following effects on the mathematics curriculum.

- Mathematics at school became descriptive - exact definitions and proofs were largely omitted.
- Geometry was neglected.
- Computations were performed by calculators and numbers and not on a more advanced level.

Students also experienced difficulties when moving from elementary school mathematics to secondary school mathematics and especially to high school mathematics. Little has been done to ease this friction.

Matematiikkalehti



Rational numbers	1981	2003
$26 + 17 =$	98.5%	89.8%
$(1/2) \cdot (2/3) =$	56.4%	36.9%
$(4/3) \cdot 5 =$	66.3%	44.4%
$(1/6) \cdot (1/2) =$	56.5%	28.3%
$(1/5) : 3 =$	49.2%	27.5%
$1278/2 =$	55.1%	36.8%

Algebra	1981	2003
$10^3 \cdot 10^2 =$	72.5%	43.3%
$x^4 \cdot x^5 =$	71.7%	47.3%
$(59^2)^3 = (59^3)^2$	61.1%	31.7%

A program for raising the level of student achievement in secondary school mathematics

by Frank B. Allen

- We must take a balanced view of the **role of problem solving** in school mathematics, lest our preoccupation with it causes us to **fragment and distort** the very mathematics that makes problem solving possible.
- Problems are the life blood of mathematics. But we must not fail to convey to our students that the body of mathematics is given structure and coherence by the bones and sinews supplied by definitions, postulates and proof. Make no mistake, **a person's problem solving ability depends on how much mathematics he understands**. Moreover, one of the principal objectives of problem solving in high school is to **inculcate a better understanding of the basic mathematical theory**. It is this understanding that will enable the student to deal with problems that are today unforeseen and unforeseeable. A student who solves a problem has devised a key that will open a specific lock. **A student who understands the mathematical theory underlying his solution has a master key that will open many locks**. Each problem should be placed in its proper mathematical context by citing the principles used in its solution.
- We must also try to develop a little more confidence in the idea that **mathematics is interesting for its own sake, and that a problem can be interesting, challenging and instructive without being obviously attached to some *real world* application**. These ideas open the door to an appreciation of the recently much neglected cultural and aesthetic values of mathematics.

What to do about Canada's declining math scores, by Anna Stokke

- Equipping students with strong problem-solving skills is an important goal of math education, but Canadian curricula and prominent resources ignore what research in cognitive science reveals about how problem-solving skills are acquired. Experienced, effective problem solvers store organized techniques in long-term memory, which allows them to categorize new problems and implement effective strategies to solve them (Sweller and Cooper 1985). The best way to ensure that students are well positioned to solve new problems is to provide them a library of knowledge and techniques and to teach thinking skills through direct instruction (Hattie and Yates 2014)
- A large body of evidence shows that direct instruction through worked examples followed by practice with problems similar to the worked examples respects working-memory limitations and improves problem-solving performance (Paas and van Gog 2006; Sweller and Cooper 1985). Gradually increasing the difficulty level of worked examples and practice problems results in the ability of students to transfer problem-solving skills to new situations. However, **when students are presented with problems that they do not have the techniques to solve without reference to worked examples, they might struggle for long periods and learn little** (Kirschner, Sweller, and Clark 2006). **This also has an obvious negative effect on students' confidence.**
- As well, discovery-based learning does not lead to a better understanding of concepts or a higher quality of learning than direct instruction. On the contrary, Klahr and Nigam (2004) find that direct instruction results in much more learning than discovery-based instruction, and that students who learn in a direct instruction environment are no less proficient at translating learning to new situations. **A particularly disturbing finding, from a number of studies, is that low-aptitude students perform worse on post-test measures after receiving discovery-based instruction than they do on pre-test measures.** In other words, discovery-based instruction might result in **learning losses and widen the gap** between low- and high-performing students (Clark 1989)

Section I

Engaging Students in Playful, Feel-good “Tasks” and Convoluted Exercises, Reform Math Fails to Help Them Develop Abstract Reasoning and Symbolic Manipulation Abilities, Rendering Algebra a Formidable Subject for All.

CMF 2021

Algebra is often taught through **symbols and symbol manipulation**, but research from neuroscience shows that students benefit from approaching content in different ways. **Algebra that is approached visually also enables students to see mathematics as a creative and connected subject.** Games are a powerful means of engaging students in thinking about mathematics. **Using games and interactives to replace standard practice exercises** contributes to students’ understanding as well as their affect toward mathematics.

The Evolution of Education

1970



Calculate the surface area of the object.

1985



Calculate the surface area of the rectangle.

2000



Calculate the surface area of the rectangle, multiplying the length by the width.

2010



Choose the correct answer.
What is the surface area of the rectangle?
[] 4000
[] 600
[] 800000

2015



Choose the correct answer.
What is the surface area of the rectangle?
[] Michael Jackson
[] Canada
[] 600
[] Breakfast

2018



Color the rectangle with the color you prefer.

edugog.com

Method A:

$$\begin{array}{r} 423 \\ - 195 \\ \hline 300 \\ - 90 \\ \hline 230 \\ - 5 \\ \hline 228 \end{array}$$

> 230

Examples of the “big idea,” “deep thinking” math from the 2021 CMF.

Method B:

$$\begin{array}{l} 423 - 195 \\ 423 - 100 = 323 \\ 323 - 90 = 233 \\ 233 - 5 = 228 \end{array}$$

5536 ÷ 82

$$\begin{array}{r} 82 \overline{) 5536.0} \\ \underline{3280.0} \\ 2256.0 \\ \underline{1640.0} \\ 616.0 \\ \underline{410.0} \\ 206.0 \\ \underline{164.0} \\ 42.0 \\ \underline{41.0} \\ 1.0 \end{array}$$

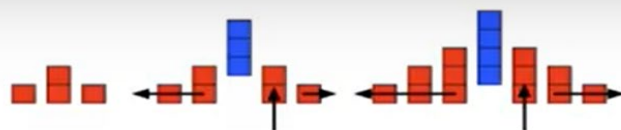
40.0
20.0
5.0
2.0
0.5
67.5

answer: 67.5

The raindrop method



The parting of the red sea



The bowling alley method



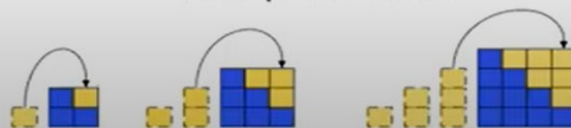
Triangular growth



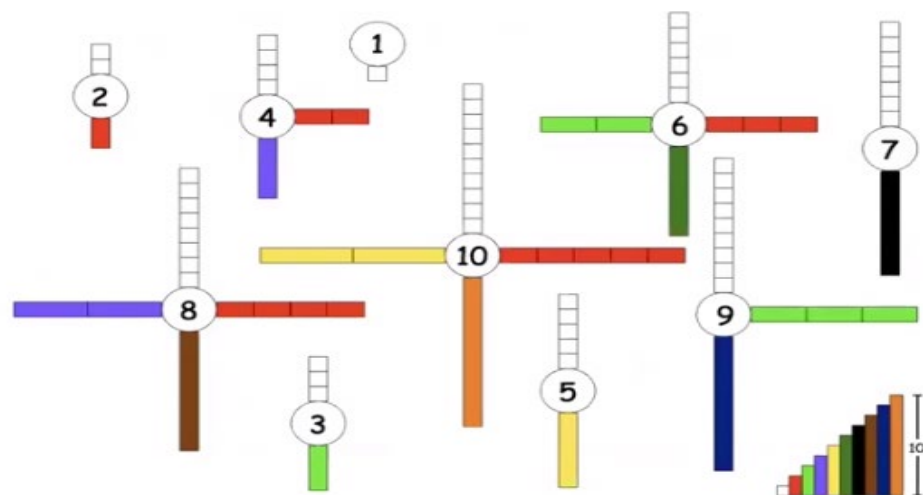
The volcano method



The square method



What do you notice?
What do you wonder?
What is going on in this data visualization?



Youcubed Data Talk
One Through Ten

One of our favorite manipulatives are Cuisenaire Rods. Here we've created a story of the numbers one through ten.

Such “engaging,” “visual,” “rich,” “multi-dimensional,” “collaborative,” “authentic,” “low floor, high ceiling” open tasks do little to grow real math knowledge and abstract reasoning ability for students.

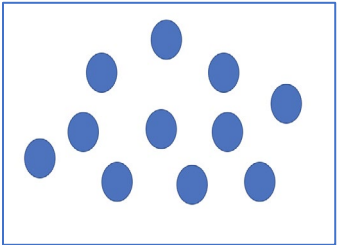
Fuzzy Math is cognitive child abuse!



CMF 2021 Sample Exercises:

Primary:

- You have a collection of objects and your friend gives you 6 more. How many do you have and **how do you know?** **Explain your reasoning using words, pictures and numbers.**

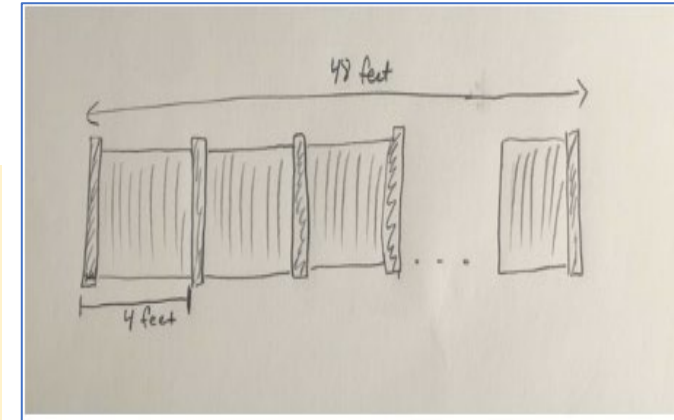


“Simple concepts are made artificially intricate and complex with the pretense of being deeper -- while the actual content taught was primitive.”

-- Marina Ratner

Upper elementary:

- You have a 48-foot-long fence made up of four-foot panels. How many four-foot panels are there? **How do you know?** Write a number sentence showing the calculation needed for this question. **Fully explain how your number sentence models this situation.**



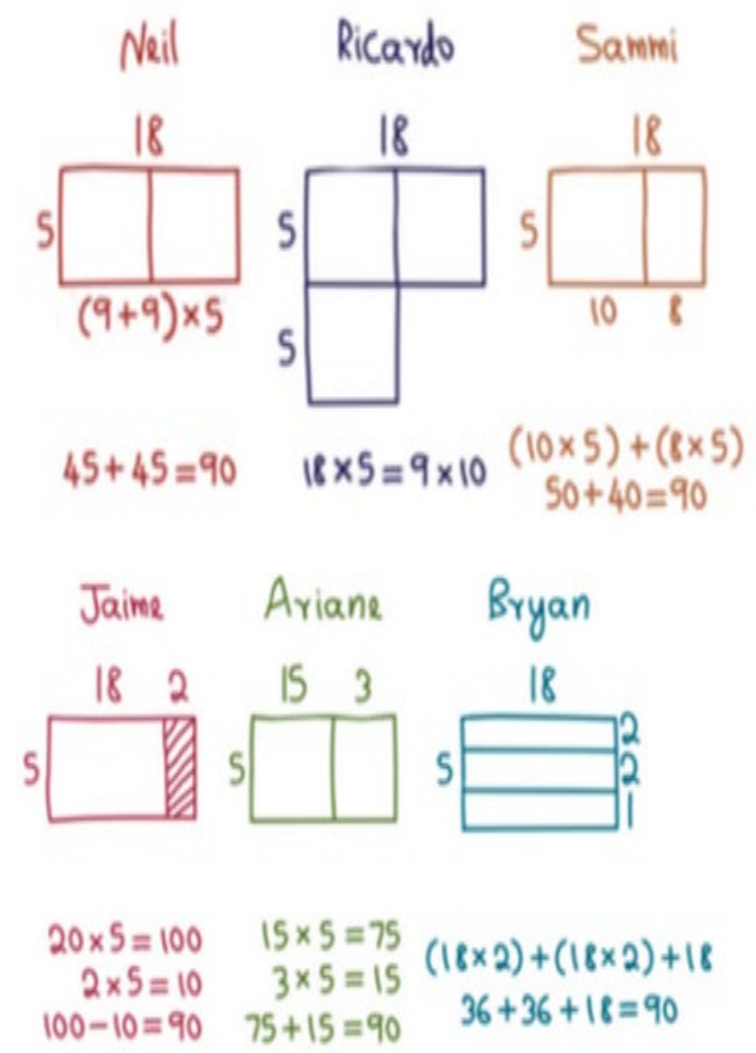
High School:

- $F(x) = 3x + 2$ where the domain is over the interval $[0, 7]$. Graph the function and include a table of values showing the integer ordered pairs. **Write a story** that might be modeled by this function. Explain how your story models the function.

Middle School:

- A point is located at -17 on a number line. If you add 8 to -17 and move the point, where will it be located? Draw the number line showing the movement and write a number sentence that represents the movement of the point. What whole number is between? **Make a convincing argument proving how you know. Explain your reasoning fully.**

Work out 18×5 and show a visual solution



from Jo Boaler. Mathematical Mindsets (2016)

Making Math Education Even Worse, by Marina Ratner

This requirement of visual models and creating stories is all over the Common Core. **The students were constantly told to draw models to answer trivial questions**, such as finding 20% of 80 or finding the time for a car to drive 10 miles if it drives 4 miles in 10 minutes, or finding the number of benches one can make from 48 feet of wood if each bench requires 6 feet. A student who gives the correct answer right away (as one should) and doesn't draw anything loses points.



Here are some more examples of the Common Core's **convoluted and meaningless manipulations of simple concepts**: "draw a series of tape diagrams to represent $(12 \text{ divided by } 3) \times 3 = 12$, or: rewrite $(30 \text{ divided by } 5) = 6$ as a subtraction expression."

This model-drawing mania went on in my grandson's class for the entire year, leaving no time to cover geometry and other important topics. While model drawing might occasionally be useful, **mathematics is not about visual models and "real world" stories**. It became clear to me that the Common Core's "deeper" and "more rigorous" standards mean replacing math with some kind of illustrative counting saturated with pictures, diagrams and elaborate word problems. **Simple concepts are made artificially intricate and complex with the pretense of being deeper -- while the actual content taught was primitive.**

What to do about Canada's declining math scores, by Anna Stokke

To summarize, there are three main difficulties with Canadian math curricula. One is the explicit **overemphasis on hands-on materials and models, which actually might hinder learning**. Another is the stress on a **multiple-strategy approach, which is time consuming, results in the overburden of working memory and the failure of students to master efficient techniques**. A third difficulty is that important concepts are introduced too late.

Provincially approved textbooks reinforce **convoluted techniques** such as drawing pictures to solve division problems such as $744 \div 6$, and multiple strategies to teach single-digit multiplication problems such as 7×8 .

Share story, by [Seattle Times staff](#)

- **It's called reform math, discovery math, constructivist math, fuzzy math. I think of it as new-age math**, and believe it is one reason why last year nearly half the 10th-graders in Washington public schools failed the mathematics portion of the high-school graduation test. It is also one reason American kids do so poorly when measured against kids from Europe and East Asia.
- New-age math, which is used in most schools today (including many private schools), came packaged with a garden basket of fragrant thoughts. "It was **hands-on**," recalls Seattle math teacher Martha McLaren. Make math fun. Small groups. Kids learning to work together, to 'appreciate the differences.' It was all going to be somehow more democratic." **It was better for girls and immigrants, who maybe didn't learn in such a "linear" way.**
- Linear it is not. One of the leading new-age series, TERC's "Investigations," leads the sixth-grade student to scissor out parts of a disk and paste them over other parts. The book tells the student, he has discovered the number pi. The lesson does not require the student to solve any problems with pi. It does not list the formula $c=2\pi r$. Instead, it prances on to a lesson about how to estimate the area of a baby's hand by counting squares on graph paper.

Why students fail calculus, by Stanley Ocken

This paper is in part a commentary on visions and perspectives of today's mathematics educators. In large part, new curricula are based on a “**constructivist**” approach to childhood education, articulated in the 1989 Standards of the National Council of Teachers of Mathematics.

That influential work describes sample topics from a constructivist curriculum and offers detailed accounts of the experiences of K-6 teachers who participated in an NSF sponsored teacher training program. One teacher comes to "realize how useless most things are out of context. How nothing makes sense mathematically if there's no concrete picture (even in one's mind) of what it is we're talking about." Another opines: "there seems to be no middle road between understanding and applying formulas." In the entire text, there is not a single instance of algebraic notation being met on its own terms. **Every fraction, every number, and every operation on numbers is reduced to pictures. Every instance of "understanding" is visual (drawing diagrams) or physical (using number sticks and other manipulatives), to the virtual exclusion of mathematical notation.** Symbols are viewed as obscurantist. "Understanding" and "constructing meaning" are synonymous with modeling. **Teachers are inculcated with a belief that algebraic manipulation can and should be avoided.**

The historical development of calculus illuminates the present discussion. It is generally understood that Newton and Leibniz, working independently, developed the main ideas of calculus. However, much less well known is that Isaac Barrow, Newton's professor, has prior claim on many of the foundational concepts of the subject. However, Barrow's name is obscured from history books because he failed to develop mathematical notation that would facilitate working with those foundational ideas and applying them to the solution of real world problems. In short, **Barrow knew the concepts, but he didn't have the right symbols and algebraic methods to work with.**

Why students fail calculus, by Stanley Ocken

- Today's calculus students are exposed to and are expected to master the symbol-rich and algebra-based methodology of Newton and Leibniz. It is their version of calculus, achieved by the reduction of geometrical ideas to algebraic notation, that has permitted the study and solution of significant problems, ranging from the motion of planets to the study of magnetic and electric fields, to the configuration of electrons orbiting the atomic nucleus, to the sophisticated mathematics demanded by medical imaging technology. All of these applications are completely intractable without the use of algebraic notation and techniques. Indeed, Newton's Principia Mathematica opens with the seemingly megalomaniacal statement: "Herein I explain the system of the world." That his grandiose claim is not an exaggeration is due in large part to his successful use of notation and formal methods to represent and quantify the problems of mathematical physics.
- Today's and tomorrow's college students need to walk into their first calculus lecture with well honed formal and symbolic skills. Insofar as K-12 curricula and educators de-emphasize algebraic techniques and denigrate algebraic formalism, they will deprive high school graduates of the opportunity to pursue mathematics-based careers.
- Understanding and symbol manipulation skill are closely related.
- In any case, the new programs seem to ignore the fact that basic computation skills are necessary in problem solving. If students lack the basic tools to yield correct results, concepts will not help. Consequently, the new programs do not appear to produce better problem solving skills as claimed.

Why students fail calculus, by Stanley Ocken

- From a pragmatic perspective, **a reasonable albeit imperfect measure of students' understanding of algebra is their ability to perform symbol manipulation flawlessly**. While one occasionally encounters an idiot savant in algebra, it is the author's firm but unsubstantiated belief that **students who consistently perform algebra without errors are prima facie demonstrating an internalized understanding of definitions and decision procedures**. Even for these students, it would be beneficial to articulate their understanding as suggested above.
- If students don't learn the long division algorithm for whole numbers, they won't be able to understand the long division algorithm for polynomials when they reach that topic in high school algebra. If they don't know long division of polynomials, they won't have any feel for techniques of integration in elementary calculus or for Laplace transforms that are introduced in more advanced courses. Even if hand computation of the answer to a division problem is no longer important, **the algebraic patterns and processes that lead to the answer are very important indeed**.
- K-12 students headed for calculus need to practice algebra **because all advanced science texts speak a universal language that is written with variables and is punctuated with subscripts, superscripts, summation signs, and lots of complicated notation**. If they're not comfortable with writing algebra, they won't be comfortable reading it. If they haven't devoted significant effort to hands-on manipulation of rational expressions, they will be in a total fog by the end of the first chapter of a typical science or engineering textbook.
- Indeed, **the accelerating mathematization of formerly descriptive sciences** such as biology makes it all the more crucial for a growing cadre of students to acquire fluent algebra skills before they begin calculus.

$$\frac{1}{G_0} \frac{d\epsilon}{d\cos\chi} = \frac{\kappa_s(\mu)}{2\pi} A(z) + \left(\frac{\kappa_s(\mu)}{2\pi} \right)^2 \left(\beta_0 \ln \frac{\mu}{Q} A(z) + B(z) \right) + \mathcal{O}(\alpha_s^3)$$

$$B(z) = C_F^2 B_{\beta_c}(z) + C_F(C_A - 2C_F) B_{n_{lc}}(z) + C_F N_f T_f B_{N_f}(z)$$

$$B_{\beta_c}(z) = \frac{122400z^7 + 244800z^6 + 157060z^5 - 31000z^4 + 2064z^3 + 72305z^2 - 143577z + 63298}{1440(1-z)z^4}$$

$$- \frac{-244800z^9 + 673200z^8 - 667280z^7 + 283140z^6 - 48122z^5 + 2716z^4 - 6201z^3 + 11307z^2 - 9329z + 3007}{720(1-z)z^5} g_1^{(1)}(z)$$

$$- \frac{244800z^8 - 550800z^7 + 422480z^6 - 126900z^5 + 13052z^4 - 336z^3 + 17261z^2 - 38295z + 19938}{720(1-z)z^4} g_2^{(1)}(z)$$

$$+ \frac{4z^7 + 10z^6 - 17z^5 + 25z^4 - 96z^3 + 296z^2 - 211z + 87}{24(1-z)z^5} g_1^{(2)}(z)$$

$$+ \frac{-40800z^8 + 61200z^7 - 28480z^6 + 4040z^5 - 320z^4 - 160z^3 + 1126z^2 - 4726z + 3323}{120z^5} g_2^{(2)}(z) - \frac{1-11z}{120z^5}$$

This illustrates the real math needed in STEM fields, which is maligned as “narrow,” “lower-order thinking,” “elitist,” and “useless” by math reformers.

SLAC theorist Lance Dixon calculates the formula for the energy-energy correlation (EEC) in particle collisions. It helps researchers better understand nature’s fundamental building blocks or even discover completely new phenomena. <https://www.youtube.com/watch?v=WVC1ygsjZNc>

The following examples use the identity $(A + B)(A - B) = A^2 - B^2$ by substituting $A = \sqrt{x + h}$ and $B = \sqrt{x}$.

Then $(\sqrt{x + h} + \sqrt{x})(\sqrt{x + h} - \sqrt{x}) = (\sqrt{x + h})^2 - (\sqrt{x})^2 = x + h - x = h$.

3.5 Let $f(x) = \sqrt{x}$; $a = x, b = x + h$. Then

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h} \\ &= \frac{1}{h} (f(x + h) - f(x)) \\ &= \frac{\sqrt{x + h} - \sqrt{x}}{h} \\ &= \frac{\sqrt{x + h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x + h} + \sqrt{x}}{\sqrt{x + h} + \sqrt{x}} \\ &= \frac{(\sqrt{x + h} - \sqrt{x})(\sqrt{x + h} + \sqrt{x})}{h(\sqrt{x + h} + \sqrt{x})} \\ &= \frac{(\sqrt{x + h})^2 - (\sqrt{x})^2}{h(\sqrt{x + h} + \sqrt{x})} \\ &= \frac{x + h - x}{h(\sqrt{x + h} + \sqrt{x})} \\ &= \frac{\cancel{h}}{h(\sqrt{x + h} + \sqrt{x})} = \boxed{\frac{1}{\sqrt{x + h} + \sqrt{x}}} \end{aligned}$$

3.6 Let $f(x) = \frac{1}{\sqrt{x}}$; $a = x, b = x + h$. Then

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= \frac{f(x + h) - f(x)}{(x + h) - x} = \frac{f(x + h) - f(x)}{h} \\ &= \frac{1}{h} (f(x + h) - f(x)) \\ &= \frac{1}{h} \cdot \left(\frac{1}{\sqrt{x + h}} - \frac{1}{\sqrt{x}} \right) \\ &= \frac{1}{h} \cdot \left(\frac{1}{\sqrt{x + h}} \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \frac{\sqrt{x + h}}{\sqrt{x + h}} \right) \\ &= \frac{1}{h} \cdot \left(\frac{\sqrt{x} - \sqrt{x + h}}{\sqrt{x}\sqrt{x + h}} \right) \\ &= \frac{1}{h} \cdot \left(\frac{\sqrt{x} - \sqrt{x + h}}{\sqrt{x}\sqrt{x + h}} \right) \cdot \frac{\sqrt{x} + \sqrt{x + h}}{\sqrt{x} + \sqrt{x + h}} \\ &= \frac{1}{h} \cdot \frac{(\sqrt{x} - \sqrt{x + h})(\sqrt{x} + \sqrt{x + h})}{(\sqrt{x}\sqrt{x + h})(\sqrt{x} + \sqrt{x + h})} \\ &= \frac{1}{h} \cdot \frac{(\sqrt{x})^2 - (\sqrt{x + h})^2}{\sqrt{x}\sqrt{x + h}(\sqrt{x} + \sqrt{x + h})} \\ &= \frac{1}{h} \cdot \frac{x - (x + h)}{\sqrt{x}\sqrt{x + h}(\sqrt{x} + \sqrt{x + h})} \\ &= \frac{1}{h} \cdot \frac{\cancel{h}}{\sqrt{x}\sqrt{x + h}(\sqrt{x} + \sqrt{x + h})} = \boxed{\frac{-1}{\sqrt{x}\sqrt{x + h}(\sqrt{x} + \sqrt{x + h})}} \end{aligned}$$

These are examples of real math practices. The requisite algebraic knowledge and skills can hardly be acquired through the “engaging,” “visual,” “rich,” “multi-dimensional,” “collaborative,” “authentic,” “low floor, high ceiling” open tasks of Fuzzy Math!!

Another pop quiz:

A piece of wood was 40 centimeters long. It was cut into 3 pieces. The lengths in centimeters are $2x - 5$, $x + 7$ and $x + 6$. What is the length of the longest piece?

Only 7 percent of American eighth graders got that one right (the answer is 15 centimeters). In contrast, 53 percent of Singaporean eighth graders answered correctly.

Which problem do you want your child doing?

Sample Investigations Math Problem (5th or 6th grade suitable)

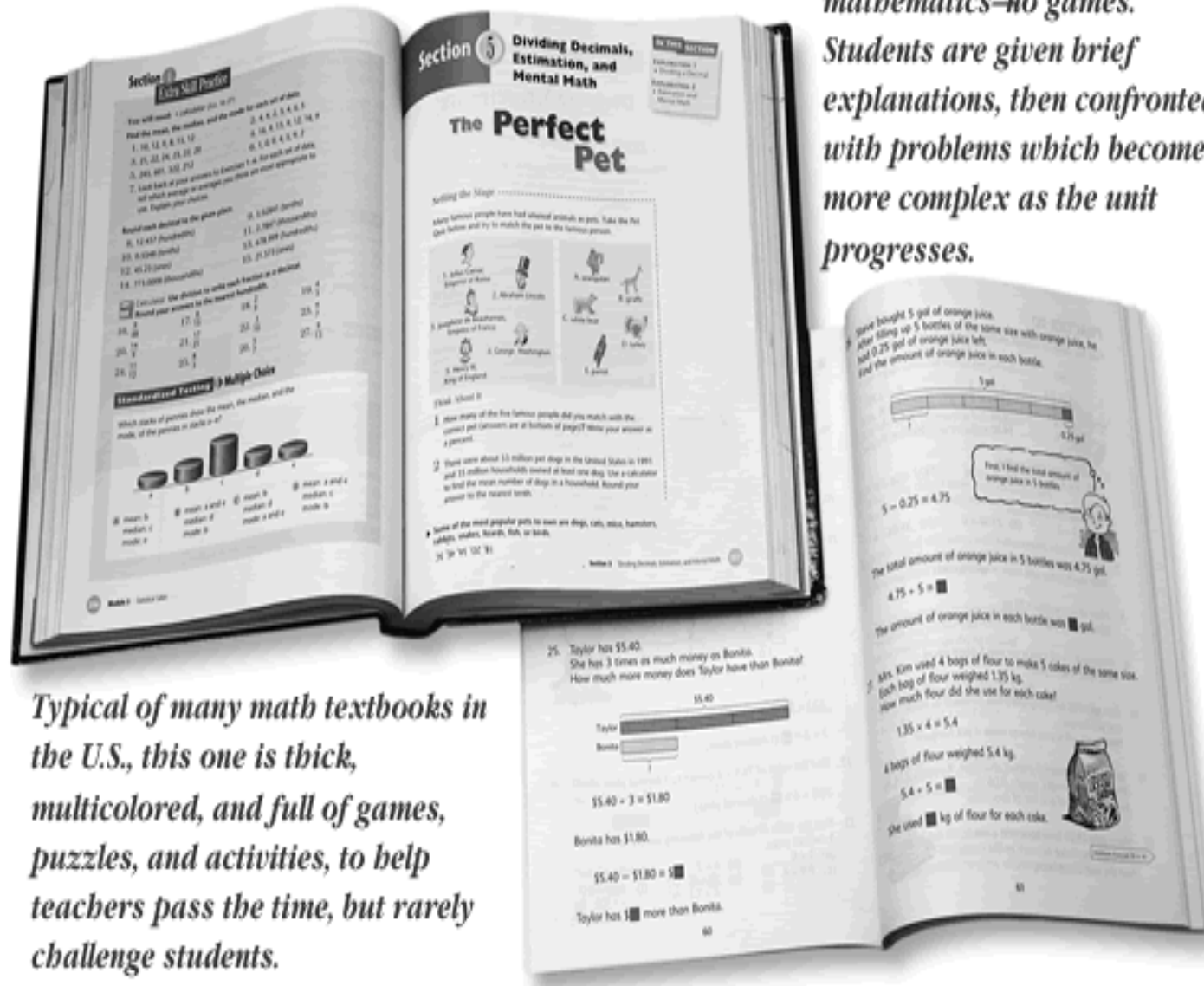
Suppose you get 6 cents for each bottle you return for recycling. For each problem show how you found your solution.

1) You have collected 149 bottles. How much will you earn?

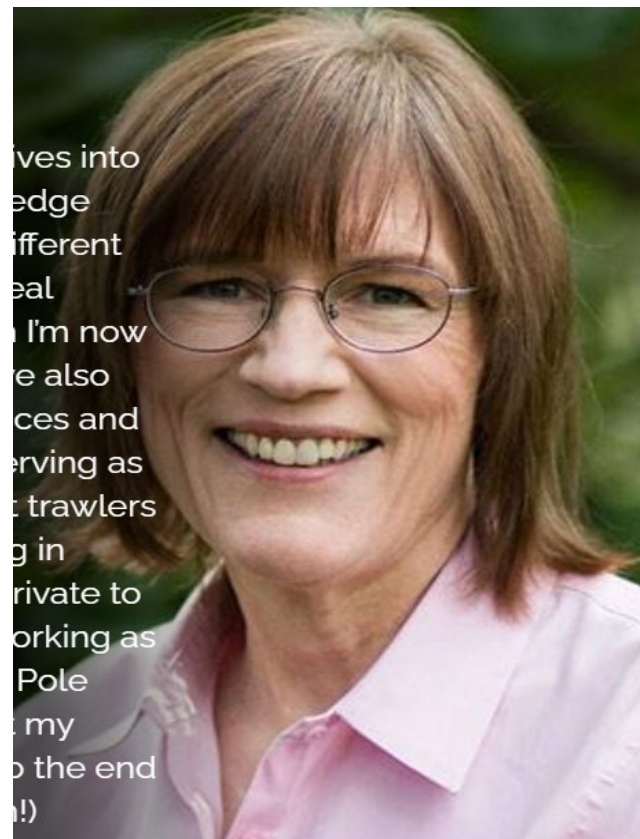
Sample Singapore Math Problem (5th grade)

Adam bought 8 note pads at \$1.45 each and 10 towels. He gave the cashier \$100 and received \$46 change. Find the cost of a towel.

Singapore Math's textbook is thin, and contains only mathematics—no games. Students are given brief explanations, then confronted with problems which become more complex as the unit progresses.

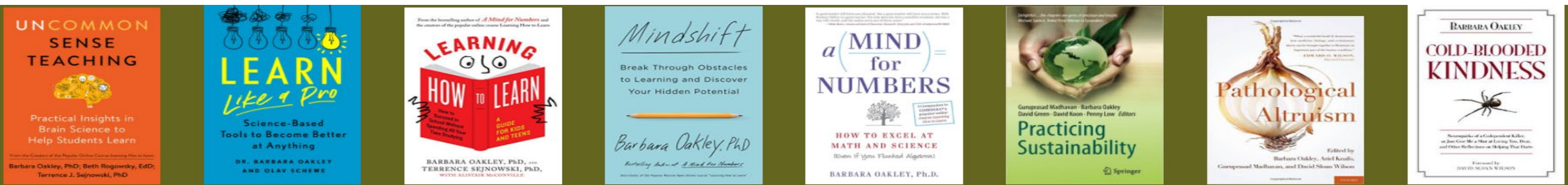


Typical of many math textbooks in the U.S., this one is thick, multicolored, and full of games, puzzles, and activities, to help teachers pass the time, but rarely challenge students.



Barbara Oakley:

- The word “rote” has a bad rap in modern-day learning. But the reality is that **rote practice, by which I mean routine practice that keeps the focus on what comes harder for you, plays an important role. The foundational patterns must be ingrained before you can begin to be creative.**
- But **today’s “understanding-centered” approach to learning math, combined with efforts to make the subject more “fun” by avoiding drill and practice, shortchanges children of the essential process of instilling the neural patterns they need to be successful. And it may be girls that suffer most.**



How I Rewired My Brain to Become Fluent in Math

The building blocks of understanding are memorization and repetition.

BY BARBARA OAKLEY

- You and your daughter can have fun throwing eggs off a building and making papier-mâché volcanoes, but the only way to create a full set of options for her in STEM is to ensure she has a solid foundation in math. Math is the language of science, engineering and technology. And like any language, **it is best acquired through lengthy, in-depth practice.**
- In fact, the more we try to make all learning fun, the more we do a disservice to children's abilities to grapple with and learn difficult topics. As Robert Bjork, a leading psychologist, has shown, deep learning involves "desirable difficulties." **Some learning just plain requires effortful practice, especially in the initial stages. Practice and, yes, even some memorization are what allow the neural patterns of learning to take form.**
- Take it from someone who started out hating math and went on to become a professor of engineering: Do your daughter a favor — give her a little extra math practice each day, even if she finds it painful. In the long run, she'll thank you for it. (And, by the way: the same applies to your son.)

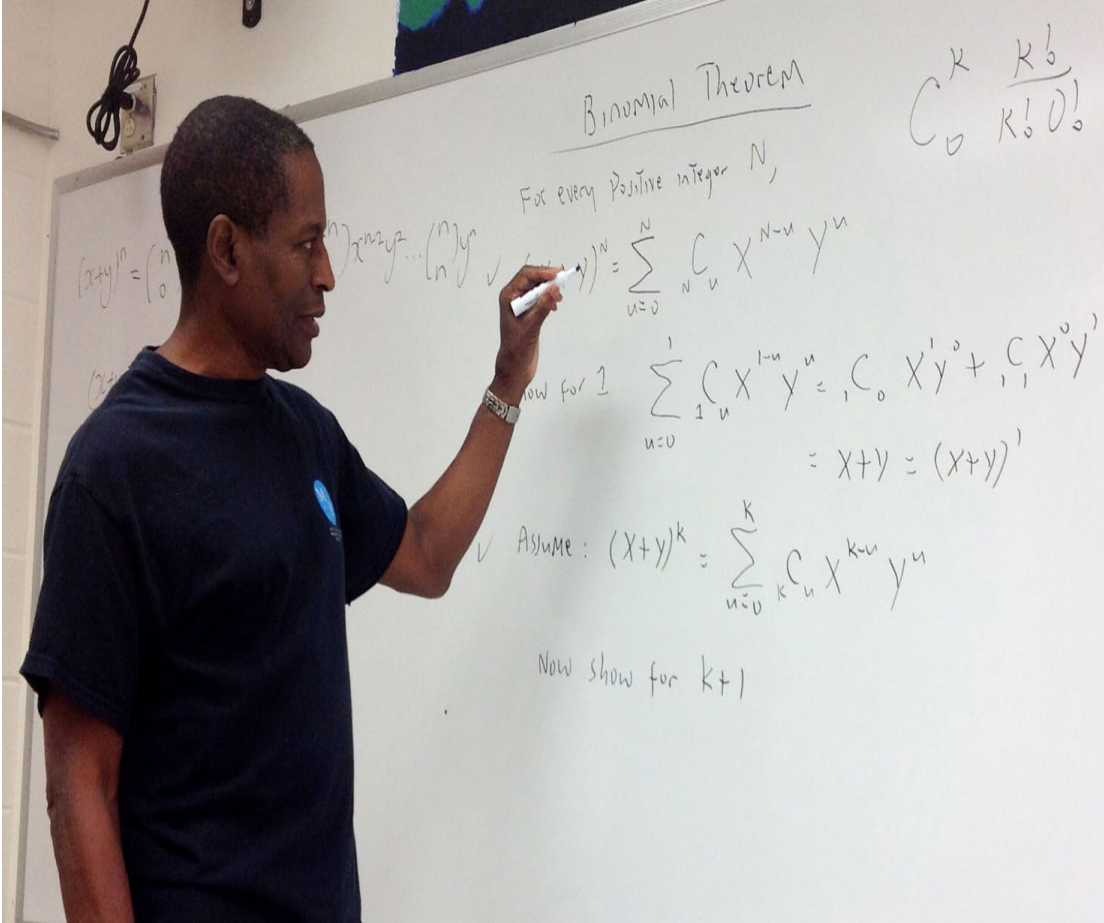
The New York Times

Make Your Daughter Practice Math. She'll Thank You Later.

The way we teach math in America hurts all students, but it may be hurting girls the most.

By Barbara Oakley

Ms. Oakley is an engineering professor and the author of a book on learning.



Vern Williams:

- Math teacher in the Fairfax County public school system for over forty years and in the gifted and talented program for over thirty years.
- The 1990 Fairfax County Public Schools' Teacher of the Year
- Received two awards for distinguished teaching from the Mathematical Association of America
- Teacher of CTY program at Johns Hopkins
- Member of the 2006-2008 **National Mathematics Advisory Panel**.

TEACHING PHILOSOPHY

Ten Myths About Math Education And Why You Shouldn't Believe Them

From [Why Johnny Can't Add Without a Calculator](#):

Math and science can be hard to learn—and that's OK. **The proper job of a teacher is not to make it easy**, but to guide students through the difficulty by getting them to practice and persevere. "Some of the best basketball players on Earth will stand at that foul line and shoot foul shots for hours and be bored out of their minds," says Williams. Math students, too, need to practice foul shots: adding fractions, factoring polynomials. And whether or not the students are bright, "once they buy into the idea that **hard work leads to cool results**," Williams says, "you can work with them."